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IS MATHEMATICS USEFUL?

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Before asking whether mathematics is useful, we need to know what "mathematics" means. One answer to this question is to say that "mathematics" means any and all of the "mathematical sciences", traditionally named "Pure Mathematics", "Applied Mathematics", and "Statistics", as well as the mathematical end of the discipline of computer science. But this answer is no help at all, for now we have to answer the questions "what is Pure Mathematics?", "what is Applied Mathematics?", and so on. Another answer might be that mathematics is what mathematicians do, and as long as we avoid defining a mathematician to be one who does mathematics, we are closer to the truth. A different answer was given by Lynn Arthur Steen, an American mathematician, who states that mathematics is the study of patterns. I would suggest a variation of this definition: one aspect of mathematics is playing with patterns, or perhaps better, structures. By this I mean activities such as looking at pictures of fractals on a computer screen, or running a computer program to "see what happens", or simply trying to guess the solution to some equations, in order to find the way to solve similar equations. So my title can be rephrased: can playing with patterns be useful?

At the beginning of this century, a ferocious debate was waged between two Oxford professors of mathematics, as to whether mathematics was useful or not. On the one hand, Professor G.H. Hardy asserted regularly that his work was not motivated in any way by real problems, and that it would never be useful (Hardy was quite a character, who stated that the work he couldn't do in the mornings was not worth doing, and spent the afternoons watching cricket, where he noticed that batsman's averages tended to stabilise towards the end of the season, and turned this into a mathematical theorem. Read his "A Mathematician's Apology" to find out more about him); on the other hand, Professor Oliver Heaviside argued that mathematics done in a vacuum was a waste of time. Curiously enough, much of Hardy's work is now applicable to the real world. For example, he worked a lot in the area of mathematics known as Number Theory, which was considered useless at the time, but which gave rise to mathematical topics now called "cryptography" and "error-correcting codes", which are in everyday use. Supermarket barcodes, for instance, have an inbuilt "check digit", and if a barcode reader reads a code incorrectly, the "check digit" is wrong (usually) and the machine makes a rude noise to ask the checkout operator to show it the barcode another time. These codes also underlie electronic data transmission, in everyday use in fax machines and automatic teller machines, as well as the passage of coded information through scrambling devices.

This example brings me to my first point: even though some mathematics is done for fun, it may well turn out to be useful. It is probably worthwhile asking why this happens. One explanation runs as follows. Psychologists tell us that play is essential to children, and through play, they learn the skills which enable them to function in the society into which they are born. Similarly, playing with patterns is the way in which we learn about "mathematical structures" which make up the world we live in. It is remarkable (a famous physicist, Eugene Wigner, called it "the unreasonable effectiveness of mathematics") that these mathematical patterns recur in different places and times, so that playing with them in one context allows us to understand them well enough to answer questions in a different context.

Let us consider some examples of how mathematics is useful. Mathematics is very pervasive, so to limit the number of examples, I shall describe topics in which research is being carried out by my colleagues at UNSW.

1) Statistics is used in almost every faculty of the university: linguists, medical researchers, geographers and engineers all use statistics. It is curious that statistics started out as play – the French gambler Pascal started to ask himself questions about gambling. Questions like "If a coin comes up 6 times heads in 10 tosses, is it loaded?" or "If a coin comes up 600 times heads in 1000 tosses, is it loaded?" meant very little to Pascal, who was so wealthy he could afford to lose at the gambling table, but the methods developed to answer these questions are also useful to answer questions like "Is drug A more effective than drug B at curing disease C?" or even "Was the play X written by William Shakespeare?"

2) Mathematics has always been associated with the physical sciences and with engineering, but the recent development of very powerful computers has strengthened these

links. In the last thirty years, two bridges in Melbourne have collapsed (one during construction) due to incorrect calculations made by the design engineers. We hope that henceforth designers will make use of the resources of the new Centre for Advanced Numerical Computation in Engineering and Science and avoid such disasters in the future.

3) Mathematics is becoming important in Environmental Sciences. Whether one likes it or not, fisherman will continue fishing, probably with ever-improving technology, and there is a risk that some types of fish will be fished out completely. Population dynamics studies such questions, and hopefully will help us to avoid disasters such as the "Peruvian anchovy catastrophe", in which Peru's fisherman took too many anchovies and depleted irrevocably the population. (The word catastrophe here is used both in a literal and a technical sense – a branch of mathematics called catastrophe theory is interested in how small changes can produce very drastic results). Another important area is bushfire research (at ADFA, in Canberra): understanding bushfires is important both from the point of view of environmentalists (how big must a national park be to be "sustainable") and users such as farmers and foresters. Finally, Oceanography, understanding the seas, is useful in management of tourism on the Great Barrier Reef or of fisheries, as well as in determining the effect of the new sewage outfalls off Sydney. And to return to an earlier theme, a look at Oceanographers in their wet suits will convince most people that there is play in Oceanography as well as practical application.

4) Mathematics is increasingly a tool of the financial sector. Many mathematics graduates now choose employment in banks or insurance companies, helping them make their financial decisions, and the Federal Government's superannuation policies mean that this sector will continue to expand. I spent much of 1991 working at AMP on investment strategies, and some of my colleagues work on pricing financial "derivatives" (options). Curiously enough, my work led me to try to solve some simultaneous equations, and I found that a standard method for finding solutions (Newton's method), which involves an initial "guess" at a solutions, and then generates better and better approximations, behaves in a curious way. Depending on the initial guess, one can get a completely different solution, and the solution one arrives at depends very oddly on the starting point. This was evidence of chaos in action (and perhaps says something about the financial world); one can

see pictures of similar phenomena in James Gleick's book *Chaos*. Another anecdote will also illustrate how odd mathematical patterns crop up unexpectedly: my option-pricing colleagues use formulas which arise in the very abstract topic of harmonic analysis on Lie groups, which is known to have connections to physics, but until this recent work, appeared to be completely unrelated to financial mathematics.

This discussion of financial mathematics brings me to my next topic: the Australian economy. Understanding economics requires understanding some mathematics, but very little mathematics is required to understand that if South-East Asian workers are prepared to make shirts for 50c per day, and Australian workers make shirts for $50 per day, then Asian-made shirts will cost less than Australian-made shirts. This fact underlies our "balance of payments crisis": we import more than we export. Possible solutions to this problem include:

a) processing raw materials before we export them. Thus we should process wool before we send it overseas, and "add value" to it. Would you believe that mathematics even gets involved here – the Wool Board supports some research into textiles at UNSW where the mathematics is pretty fancy.

b) We should try to export "niche" items of high technology, such as computer programs, bionic ears, and pacemakers. (These are all areas in which Australia is quite successful). Small items reduce the disadvantage of being far from the big markets of Europe and North America, and with items made in small numbers we do not run into problems competing with overseas companies with bigger, brighter, and better production lines. However, as Ed David, chairman of Exxon Research in the USA put it, "high technology is mathematical technology". Mathematics underpins most new technology, often in surprising ways.

My final point is that this country has a choice of "following the Argentinian road" and "becoming a banana republic", or developing small, clean, high technology industry. More, and better trained, mathematicians are essential to the long term health of Australia. Mathematics is not just useful, it is fun and it is essential.