PROBLEM SECTION

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year or form. Solutions of these problems will appear in the next issue of Parabola; your solution(s) may be used if they are received in time.

Q.872 A flea on the number line jumps from the point a to the point b, given by $a + \frac{1}{b} = 1$. If the flea keeps jumping according to the same rule, show that it comes back to its original position eventually. In how many jumps? (Assume $a \neq 0,\ a \neq 1$.)

Q.873 In my laundry, the ceiling is laid out in squares, like the number plane. There is a light at the centre, which we will think of as the origin. A moth flits about on the ceiling according to the following rule:

If it is at (x, y), it flits to $(y, y-x)$. Show that it eventually comes back to where it was. How many flights does it make? What shape of path does it take? Consider the brightness of the light at (x, y), given by $b = 1/(x^2 - xy + y^2)$. What extra information does this give you about the path the moth follows?

Q.874 A person can climb a staircase one or two steps at a time. In how many different ways can (s)he climb a staircase of 10 steps? of 100 steps? of n steps? (There are 5 ways of climbing a staircase of 4 steps: $1,1,1,1; 1,1,2; 1,2,1; 2,1,1; 2,2$.) What if (s)he climbs 2 or 3 at a time?

Q.875 A sequence $\{u_n\}$ is given by the formula

$$u_n = Ar^n + Bs^n$$

for fixed numbers A, B, r, s. Show that for every n,

$$u_{n+2} = (r+s)u_{n+1} - rsu_n$$

Given that $u_1 = 1,\ u_2 = 2,\ u_3 = 3,\ u_4 = 5$, find A, B, r, s and hence find a formula for u_n.

Q.876 Given a set of $2n+1$ numbers in arithmetic progression, in how many ways can 3 distinct numbers also in arithmetic progression be selected?

Q.877 A regular pentagon has all its diagonals drawn in; there is a small pentagonal region in the centre. What is the ratio of the area of the smaller pentagon to the larger? Try to give your answer in terms of $\sqrt{5}$.

Q.878 Consider the parabola $x^2 = 4ay$, and let P, Q, R be three points on it. The three tangents to the parabola at P, Q, R meet in pairs at points S, T, U. Prove that the area of ΔSTU is half that of ΔPQR.

Q.879 A four-by-four block of one-by-one squares has a number in each of the sixteen squares, in such a way that the numbers in each row, each column and each diagram have the same sum, k say. Show that the numbers in the four corners sum to k.

Q.880 The number x satisfies the equation

$$x^2 = \sqrt{2}x + \sqrt[3]{3}.$$

Show that x satisfies a polynomial equation with **rational** coefficients. (We call such a number "**algebraic**".)

Q.881 If you have seventeen points in space and each pair is joined by an interval coloured either red, green or blue, show that there is a red, a green or a blue triangle.