In the last issue we posed the following question.

"Two companies, A and B, offer starting salaries of \$20000 but A gives an annual rise of \$2000 whilst B gives a half-yearly rise of \$500. Which company should we work for if we wish to maximize our income?"

The question is discussed in Morris Kline's book **Mathematics and the Search for Knowledge** (OUP 1986). He claims that most people would choose A even when it is understood that B is to earn \$10500 in the 2nd half of his first year. There are other interpretations of how B's income grows (see Rod James' article) but the question still supports Kline's assertion that our intuition is not always reliable. The following note by George Harvey solves the general question with the interpretation of Kline.

SALARY OPTIONS

George Harvey*

(1) Under what condition is an increment of \$$b$ paid k times a year [Option (B)] more advantageous than an annual increment of \$$a$ paid yearly [Option (A)]?

(2) When the condition in (1) is satisfied, what is the least number of years before Option (B) establishes its superiority?

[Assume that increments are effective immediately. It is obvious that the merits of the options are independent of current salary which we may therefore assume with loss of generality to be zero].

Notation: A_n, B_n = salary in nth year under option $(A), (B)$ respectively.

(1) **Salary in nth year** $A_n = na$

$$B_1 = (1 + 2 + 3 + \cdots + k)b$$

$$B_2 = [(k+1) + (k+2) + (k+3) + \cdots + (k+k)]b$$

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$$B_3 = [(2k+1)+(2k+2)+(2k+3)+\cdots+(2k+k)]b$$

....

$$B_n = [((n-1)k+1)+((n-2)k+2)+((n-1)k+3)+\cdots+((n-1)k+k)]b$$
$$= \frac{1}{2}k[((n-1)k+1)+nk]b = \frac{1}{2}k[2nk-(k-1)]b = bk^2n - \frac{1}{2}bk(k-1)$$

$\therefore B_n - A_n = (bk^2 - a)n - \frac{1}{2}bk(k-1)$,

which increases with n if and only if $bk^2 > a$, (1)

and $B_n > A_n$ if and only if $(bk^2-a)n > \frac{1}{2}bk(k-1)$

i.e. if and only if $n > \dfrac{\frac{1}{2}bk(k-1)}{bk^2-a}$ (2)

(2) **Salary earned in n years**

$$\Sigma A_n = (1+2+3+\cdots+n)a = \frac{1}{2}n(n+1)a$$
$$\Sigma B_n = (1+2+3+\cdots+nk)b = \frac{1}{2}nk(nk+1)b$$

$\therefore \Sigma B_n - \Sigma A_n = \frac{1}{2}nk(nk+1)b - \frac{1}{2}n(n+1)a$, which is positive if and only if

$n > \dfrac{a-bk}{bk^2-a}$ [since $bk^2 - a > 0$, by (1)] (3)