PROBLEM SECTION

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year or form. Solutions of these problems will appear in the next issue of Parabola; your solution(s) may be used if they are received in time.

Q.882 A triangle is divided by one straight line into two parts which are similar to each other. Prove that the triangle is isosceles or right-angled (or both).

Q.883 A number consists of the nine digits $1, 2, \ldots, 9$ once each (in some order). The last digit is 5. Prove that the number is not a square.

Q.884 On a clock face, six adjacent numbers are left untouched and the other six are rearranged so that all around the clock face, every pair of consecutive numbers adds up to a prime number. What is the final arrangement of numbers?

Q.885 Prove (without extensive calculations!) that

$$\frac{31}{45} < \frac{1000}{1001} \times \frac{1002}{1003} \times \frac{1004}{1005} \times \cdots \times \frac{1992}{1993} < \frac{23}{31}.$$

Q.886 Find three prime numbers a, b, c, all different, such that

$$a^2 + 37ab = c^3 + 1656.$$

Q.887 Last year was a busy one for my family, with six of my brothers and sisters having children. Writing myself a timetable for buying niecely and nephewly birthday presents, I noticed some odd facts about the six dates. The difference (in days) between two consecutive birthdays was always the same, and this difference was a prime number. No two children were born in consecutive months, and no two were born on the same date in different months. One of the birthdays was on August 8. When were the others?

Q.888 Alexander, David, Esther, Jacinda and Simon all received different marks in the maths test which was held unexpectedly last week. In the following, students who made correct statements invariably had obtained higher marks than those who made incorrect statements.

Simon:	Alexander and Esther gained the top two places.
Jacinda:	No, what Simon just said is wrong.
David:	I was ranked in between Simon and Jacinda.
Alexander:	Jacinda came second.
Jacinda:	I scored fewer marks than Esther.
Esther:	Exactly three of the previous five statements are correct.

Find the order in which the students finished.

Q.889 Find all positive integers m such that if

$$(1993+m)^{1993}$$

is expanded by the Binomial Theorem, two adjacent terms are equal.

Q.890 If n is a positive integer, we define $N(n)$ to be the number of ways of writing

$$n = x_0 + 2x_1 + 2^2x_2 + 2^3x_3 + \cdots$$

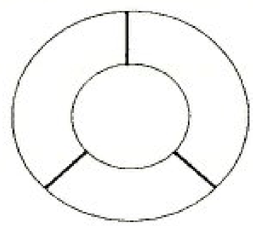
where $x_0, x_1, \ldots$ may take the values $0, 1, 2$ or 3. For example, $N(9) = 5$ since

$$\begin{aligned} 9 &= 1 + (2\times 0) + (2^2\times 0) + (2^3\times 1) \\ &= 1 + (2\times 0) + (2^2\times 2) \\ &= 1 + (2\times 2) + (2^2\times 1) \\ &= 3 + (2\times 1) + (2^2\times 1) \\ &= 3 + (2\times 3) \end{aligned}$$

are the five ways of writing 9 in the given form. Find a formula giving $N(n)$ in terms of n.

Q.891 Andrew is given a bag of lollies by his parents and told to share them with his little sister Becky. The number of lollies in the bag is not known, but is somewhere from 1 to 100. Being just a little bit greedy, Andrew shares out the lollies according to the scheme "one for you, two for me, three for you, four for me, $\cdots$" Any leftover lollies at the end go to the person who would have received them anyway. Thus if there are eleven lollies, then Becky gets one, Andrew gets two, Becky gets three, Andrew gets four, and Becky gets the last one. After all the lollies are shared out, how far ahead, on average, can Andrew expect to be?

Q.892 **The interplanetary map-colouring problem.** You may have heard of the Four-Colour Map Theorem (until recently the Four-Colour Conjecture). Given a map divided up into countries, it is desired to give each country a colour in such a way that any two countries with a common border bear different colours. (A single point does not count as a common border.) How many colours are needed? It is easy to see that four colours may be necessary, as for example in the following map.



It has been known for a long time that five colours are enough to colour any map whatsoever, but was shown only recently (and somewhat controversially) that in fact you can do without the fifth – four colours are (sometimes) necessary, and (always) sufficient.

Now consider the following problem. A certain race of beings lives on two planets. Each nation owns one connected piece of territory on each planet. It is desired to colour a map (pair of maps?) of the two planets according to two rules. First, (as above) two countries with a common border must bear different colours; second, each nation must have its two territories (one on each planet) coloured with the same colour.

(i) Find an example of a pair of maps which requires eight colours.

(ii) (Probably difficult.) Can you find an example where nine colours are needed?