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TIME AND RELATIVITY

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Special Relativity

Common sense tells us that either two things happen at the same time or they do not. But a hyper-accurate atomic clock that is taken on a fast train moving at a constant velocity will run slowly when compared to an identical clock left at the station. If common sense is correct, which clock do you believe – or do you believe that physics is different in a moving train?

Albert Einstein's Special Theory of Relativity (first proposed in 1905) leads us to the, at first startling, conclusion that for a moving object time (and length) will change when measured by a second observer. Moving clocks run slowly; a moving ruler gets shorter; the order that distant events occur can be different. Experiments like the one suggested above have been done and show that the effects of Special Relativity are measurable: clocks taken on air trips run slowly. In this article we will look at some of the at first sight strange and mysterious effects that Special Relativity predicts.

Special Relativity begins with two postulates relating to the world of our experience and how we measure it. The first postulate seems harmless enough, and was used in a restricted form by Newton: *the laws of Mechanics are the same to any inertial observer.* Recall that an inertial observer is one for whom Newton's First Law holds: *an object on which no force acts moves in a straight line at constant speed.* For example, if you are going around on a roundabout then you are not an inertial observer as if you roll a ball away from you it follows a curved path as you measure it on your roundabout. Einstein extended this postulate to cover all physical processes, which by then included electromagnetism and optics (which by the work of James Clerk Maxwell had become a branch of electromagnetism).

The second postulate of Special Relativity also seems innocuous, and in fact when Einstein proposed Special Relativity there was some experimental evidence for it: *there*

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is an inertial observer for whom light signals in vacuum travel at a constant speed in all directions whatever the motion of the light source. This constant speed, usually called c, is about $2.988 \times 10^8 ms^{-1}$. If this postulate were not true then the motion of an object moving backwards and forwards would appear jerky as its speed keeps changing.

Combining these two postulates leads us to the apparently absurd conclusion that, in vacuum, light travels at speed c in all directions at all times according to all inertial observers (however fast they are going). This is very different to our everyday experience. If an apple core is thrown out of a moving car then the core's velocity according to a kangaroo travelling in a paddock at the side of the road is the sum of the car's velocity, the velocity with which the core was thrown out of the window and the 'roo's own velocity. Light is not like that: however fast you move your torch and however fast the person seeing it is going, the light it produces travels at the same speed.

Measuring Time

We can get a grip on how the speed of an object affects the rate of a clock using some very simple mathematics. We simplify by using the first postulate and deal with the case of only one space dimension. We draw diagrams as in figure 1, where the vertical represents time and the horizontal the one space dimension. We will perform all our measurements with light signals, as light moves at constant velocity. We also scale the axes on these diagrams so that light signals move on lines at 45°. So anyone or anything travelling at less that the speed of light will follow a line at an angle of less than 45° to the vertical.

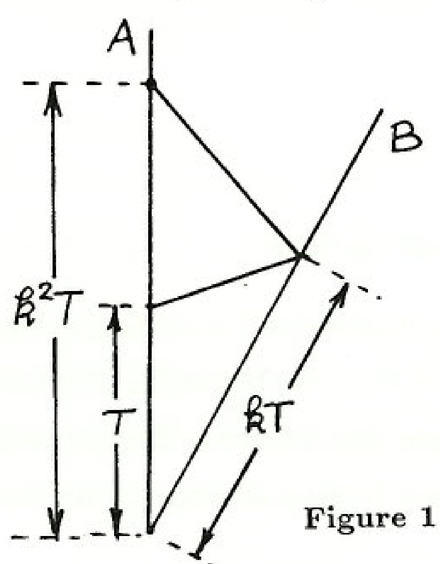


Figure 1

In figure 1 we have two observers, Alan and Beryl with Beryl moving at constant speed relative to Alan. They have identical clocks, which they synchronise to zero as they pass each other, and of course each measures time with their own clock. After time T observer Alan sends out a light signal. This reaches Beryl when her clock reads kT, say, for some real number k which must be greater than 1, as the light has had to travel some distance. Beryl immediately sends out a light signal back to Alan which must reach Alan when his clock shows k^2T. This is because according to the first postulate, Beryl's view of the universe is just as valid as Alan's. She sees time kT between passing Alan and receiving the signal (which he sent at his time T) so Alan must see time $k(kT)$ between passing Beryl and receiving her signal (which she sent at her time kT). It follows from this that Alan will assume that Beryl is at a distance $\frac{1}{2}(k^2-1)Tc$ away from him when she received the signal, as the light took $(k^2-1)T$ to get there and back and travelled at speed c. Furthermore Alan must assign the time $\frac{1}{2}(k^2+1)T$ to the moment when Beryl receives the signal, as he must assume that the light take as long to get there as to come back. If Alan attempts to make an allowance for Beryl's velocity then he would have to assign different times to Beryl's response and to the response of a third observer moving with different velocity but answering from the same place at the same time – not a useful way of measuring the universe.

Thus, according to Alan, Beryl travelled a distance $\frac{1}{2}(k^2-1)Tc$ in time $\frac{1}{2}(k^2+1)T$, and so she has speed

$$v = \frac{k^2-1}{k^2+1}\,c. \tag{1}$$

This must also be the speed of Alan as measured by Beryl. Note from this that v is always between $-c$ and $+c$: an object cannot move faster than light. The light signals would not reach it if it did, which spoils our system of measurement; such an object (if it exists) would be invisible to us.

We can solve (1) for k and find that

$$k = \sqrt{\frac{c+v}{c-v}}. \tag{2}$$

If we change v to $-v$ then k changes to $1/k$, a fact we will use later.

We further note from these calculations that the time that elapsed (according to Alan) between passing Beryl and her reception of the signal is $\frac{1}{2}(k^2+1)T$, but to Beryl this time gap is kT. So if Alan is watching Beryl doing experiments, then whenever Beryl measures a time gap of T, Alan must assign a time gap greater than T (after allowing for the travel time of light) by a factor of

$$\frac{k^2+1}{2k} = \frac{1}{2}\left(k + \frac{1}{k}\right) = (1 - v^2/c^2)^{-1/2} \geq 1 \tag{3}$$

using (2). This latter expression is usually called γ, *the gamma factor*. Equation (3) shows how time is dilated by motion: moving clocks run slower by a factor of γ. That is, the clock that Beryl is using will seem to Alan to be running slowly. Of course the situation is symmetric: Beryl will see Alan's clock run slowly with respect to her clock at the same rate.

This effect is not just an artefact of our choice of measuring systems, it is a real effect. Certain types of elementary particles called muons are created in the upper atmosphere of the earth and are detected on the ground. However, their lifetime when at rest is so short that even if they had travelled at the speed of light to pass through the atmosphere, they would have got less than one tenth of the way through before decaying – except that time dilation "keeps them young". Similar experiments done in particle accelerators with muons moving at speeds where γ is about 29 (99.88% of the speed of light) confirm the predictions of Special Relativity.

Simultaneity

We see from the above that requiring Alan and Beryl to measure time by their own clocks (a harmless enough requirement) and to use light signals to measure distances (radar would be equivalent, as radio waves are another form of the same sort of radiation) has lead us to conclude that the rate that time passes is not universal, but depends on how fast you are going. Even more remarkable is the fact that the very order of distant events can be different for different people. Consider figure 2, which show Alan and Beryl again going past each other at a time both call $T = 0$.

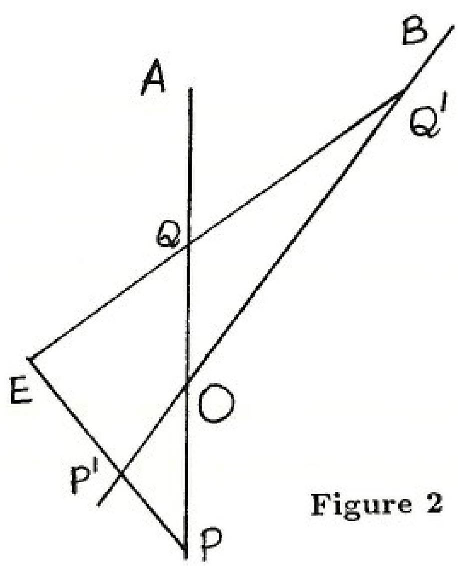


Figure 2

The event E occurs at some distance from time t_0(Q on the diagram). By the same reasoning used earlier, he must assign time $T = 0$ to the event E. So for Alan, E happened at the same time as he passed Beryl.

Now consider Beryl. If she is to do the same measurement of the time of E, she must send out a signal when Alan's signal reaches her (at P' on the diagram), and she will receive the bounced signal at Q'. We can use the k factor introduced earlier to find the time on Beryl's clock when she sends her signal and when she receives it. Assuming that Beryl moves with the same speed v before and after passing Alan we find that she measures a time gap of $(k - k^{-1})t_0$ between P' and Q', k as in equation (1). Thus according to Beryl, event E happened at time

$$\frac{1}{2}\left(k - \frac{1}{k}\right) t_0 > 0.$$

Thus for Beryl, E happened *after* she passed Alan.

Obviously, if a third observer went past Alan (and Beryl) at $T = 0$ going in the opposite direction to Beryl, that observer would measure E as happening *before* the three met. What we get from this is that if events occur at different places then the order that they occur is depends on the observer, or in the usual jargon *simultaneity is relative.*

The Twin Paradox

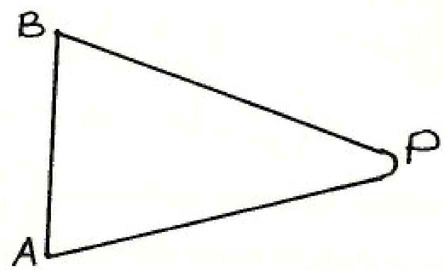
We now look at the oldest of the so-called paradoxes of relativity: the twin paradox, due to Paul Langevin (1911).

Suppose Alan and Beryl are twins, and Beryl goes off on a long journey through space at high speeds, and then returns to Alan who has been an inertial observer while she is away. According to time dilation, Beryl will have experienced much less time than Alan and will, as a consequence, be physically younger when she returns. How much younger will depend on her speed and could be calculated using γ.

Now comes the apparent paradox. According to Relativity's must basic rule Alan could just as well argue that Beryl stayed at rest, but he went on the trip: why is he older than Beryl when they meet again?

The problem here is that we are trying to apply the first postulate to a case where it cannot apply. In order to undertake her trip, Beryl must have accelerated away from Earth, and then accelerated again to come back (and an third time to land on Earth). All these accelerations mean she cannot have stayed inertial: there is no symmetry between the twins.

Neither can we argue that we could cut out the initial and final accelerations by synchronising clocks that go past each other: if one clock is to make each journey, the one that stays young must experience some sort of acceleration. It is also argued that we could keep the acceleration either very gentle or very brief, so that the youthful clock is either nearly inertial or inertial for all but a brief time. However the first idea is like claiming that a large semi-circle is nearly straight and so should have nearly the same length as a diameter. The second is, as Hermann Bondi pointed out, like the case of two drivers who go from A to B, the first in a straight line and the second in a wide deviation to P as in figure 3. Both lines are straight almost everywhere, but one is clearly longer than the other.



Two lines of nearly the same length?

Figure 3

Conclusion

What we have been discussing in this article is the effect that motion can have on the rate of a clock (or any other measurement of time). All these effects follow from the basic postulates of Special Relativity, which essentially say that non-accelerated motion has no effect on physical processes and that the speed of light in vacuum is always the same. It is important to realise that this slowing down is only noticeable as an effect relative to some other observer. If you are moving very rapidly, you do not feel any difference.