PROBLEM SECTION

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year or form. Solutions of these problems will appear in the next issue of **Parabola**; your solution(s) may be used if they are received in time.

HINT. Some of the following problems are similar to, or based on, problems from this year's School Mathematics Competition. Solutions to the competition problems will be found elsewhere in this issue of **Parabola**.

Q.893 Find all positive integer solutions of

$$x^2 - 84 = 6y + 3x - 2xy.$$

Q.894 A six-digit number was divided by a three-digit number, giving a three-digit quotient and no remainder. In the working, every even digit $(0,2,4,6,8)$ was replaced by an E, while every odd digit $(1,3,5,7,9)$ was replaced by an O. Given the result shown below, reconstruct the working. No number begins with a zero.

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             O O E
O E O  )E E O E E E
        E E E O
            E O E
            O E O
              E O E
              E O E
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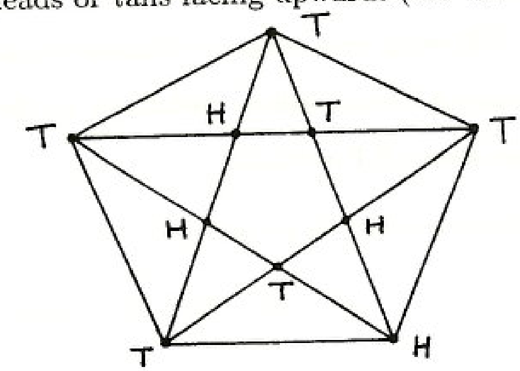
Q.895 Take a normal 8×8 chessboard and remove from it as few as possible individual squares, in such a way that no 3×1 rectangle can be placed so as to cover three squares on the remaining part of the board.

Q.896 (Adapted from a puzzle heard on ABC radio.) Take a square-based pyramid whose triangular faces are all equilateral, and a regular tetrahedron whose faces are of the same size as the triangular faces of the pyramid. Join these two solids along a pair of triangular faces. In the combined solid you will see two triangles which appear to lie very nearly in the same plane. Do they in fact lie exactly in the same plane? Prove your answer.

Q.897 Let $x = \sqrt[3]{10} + \sqrt[3]{6}$. Show that $x^3 - 3x\ \sqrt[3]{60} = 16$, and deduce (without a calculator!) that $x < 4$.

Q.898 A pentagon with all its diagonals drawn in has a coin placed on each intersection of lines, with either heads or tails facing upwards (see the diagram).



It is permitted to select any one of the ten lines in the figure, and turn over all the coins lying on that line; such a move may be performed as often as you like.

(i) Find a sequence of moves starting with the above position and finishing with all coins facing up heads.

(ii) Find a rule by which it is possible to tell merely by studying any initial position whether or not the task in (i) can be accomplished.

Q.899

(i) We have seven coins, apparently identical, of which two are heavier than the other five (and the two heavy coins weigh the same as each other). With three weighings on a beam balance, find the heavy coins.

(ii) Show that the above problem cannot be solved if we have eight coins, with two heavier than the other six.

Q.900 Consider the binomial expansion of $(x+1)^n$.

(i) If the expansion contains three consecutive coefficients such that the second and third are (respectively) twice and three times the first, find n.

(ii) If it contains three consecutive coefficients such that the second and third are respectively a times and 23 times the first, where a is an integer, find a and n.

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