PROBLEM SECTION

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year or form. Solutions of these problems will appear in the next issue of **Parabola**; your solution(s) may be used if they are received in time.

Q.903 Find all positive integers n such that $1+2+3+\cdots+n$ is a factor of $1\times 2\times 3\times\cdots\times n$.

Q.904 The average value of the coins in my pocket is 83 cents. After taking out two coins, not both the same, I find that the average has increased to 90 cents. What were the two coins? (All current Australian coins are allowable, that is, 5c, 10c, 20c, 50c, \$1 and \$2.)

Q.905 Find all solutions of

$$x^4 - 4x^3 - 2x^2 + 12x + 8 = 0.$$

Q.906 (a) All of the following.

(b) None of the following.

(c) Some of the following.

(d) All of the above.

(e) None of the above.

Q.907 In the middle of a large field a patch is fenced off in the shape of a regular hexagon. A goat outside the hexagon is tethered by a rope attached to one corner of the hexagon and having length $2+\sqrt{3}$ times the side of the hexagon. Find the total area within which the goat can graze.

Q.908 Solve in positive integers

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} = 2^m.$$

(Here the terms on the left hand side are binomial coefficients, that is, $\binom{n}{k}$ is the coefficient of x^k in $(1+x)^n$. They are sometimes written $C(n,k)$ or ^nC_k.)

Q.909 Is it possible to place the numbers 1, 2, 3, ..., 10 around a circle in such a way that the maximum sum of three adjacent numbers is (i) 16; (ii) 17; (iii) 18? If so, show how to do it; if not, explain why not.

Q.910 A list of numbers $u_1, u_2, u_3, \ldots$ is defined as follows. The first number is $u_1 = 2$, and for $n \geq 1$ we let

$$u_{n+1} = 2u_n + \sqrt{3u_n^2 - 12} \,.$$

For example $u_2 = 2u_1 + \sqrt{3u_1^2 - 12} = 4$ and $u_3 = 2u_2 + \sqrt{3u_2^2 - 12} = 14$. Prove that all the numbers in the list are integers.

Q.911 (i) Two players alternate tossing a coin. The winner is the first player to have thrown both a head and a tail at least once each. What is the probability that the first player wins?

(ii) Two Martians alternate tossing a coin. The winner is the first player to have thrown heads, tails and paws at least once each. (Martian coins have three sides, all of which are equally likely to turn up.) What is the probability that the first player wins?

Q.912 I tell Jack and Jill each a positive integer. I tell both of them that the sum of the two positive integers is either 4 or 5, and ask each in turn whether they know the other's number.

Jack:	"I don't know."
Jill:	"I don't know either."
Jack:	"I still don't know."
Jill:	"Now I know!"

How *did* she know?