PROBLEM SECTION

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year or form. Solutions of these problems will appear in the next issue of **Parabola**; your solution(s) may be used if they are received in time.

Q.931 In a large flat area of bushland there are two fire-spotting towers, one exactly 20km east of the other. A bushfire is reported as being due north-east of the western tower, and simultaneously due north-west of the eastern tower. However each of these directions could be in error by up to $1°$ either way. Find the total area within which the fire might be located

(a) by a simple approximate argument;

(b) exactly.

Q.932 Four weary explorers have to cross a bridge over a river one night. Owing to their various degrees of exhaustion, they would individually take 5, 10, 20 and 25 minutes (respectively) to cross the bridge. However, the old and rickety bridge will take only one or two people at a time. Furthermore, it is too dangerous to cross the bridge in the dark, and the expedition has only one torch. How can all four explorers cross the bridge in the least possible total time?

Q.933 (a) In how many ways can 1994 be written as the sum of (one or more) consecutive positive integers?

(b) Prove that for any positive integer n, the number of ways of writing n as the sum of (one or more) consecutive positive integers is equal to the number of odd factors of n.

(c) Deduce from (b) that a positive integer can be written as the sum of two or more consecutive positive integers if and only if it is not a power of 2.

Q.934 Prove that there is no polyhedron (solid figure bounded by plane surfaces) having 7 edges, but there is one with any number of edges greater than 7.

Q.935 A triangle has integer sides. Each side is divided into intervals of length 1 and the midpoint of each interval is marked. Prove that it is possible to draw a continuous path, linking all these midpoints and returning to its starting point, subject to the following conditions:

(i) every point is visited once, and no point is used more than once (except that the first point is the same as the last);

(ii) successive points on the path must come from different sides of the triangle.

Q.936 Find all solutions in positive integers of

$$6x^2 + 3y^2 + 6z^2 - 8xy - 8yz + 10xy = 6.$$

Q.937 An $n \times n$ chessboard has a number of beans placed on each square. The squares in the top row contain (from left to right) $1, 2, 3, \ldots, n$ beans; in the second row $n+1,\ n+2,\ n+3, \ldots, 2n$; and so on, ending with n^2 beans in the bottom right hand corner. It is permitted to select any two rows and remove from each square in one of them the number of beans in the corresponding square in the other one. For example, if one row contains 1,4,3 beans and another 2,7,4 the latter may be changed to 1,3,1; then in the next move, the first row may become 0,1,2. "Negative beans" are not allowed (for example, no change is possible on the above rows containing 0,1,2 and 1,3,1 beans respectively).

(a) After performing the above operation as many times as you wish, what is the smallest possible remaining number of non-empty rows on the chessboard?

(b) What is the minimum possible total number of remaining beans?

(c) What are the answers to (a) and (b) if we allow not only the above operation on the rows of the chessboard, but also a similar operation on the columns?

Q.938 Let α be a constant. Show that the x-axis is tangent to the curve

$$y = x - \sin x - (1 - \cos x)\tan\alpha$$

if and only if $\tan\alpha - \alpha$ is a multiple of π.

Q.939 Which of the statements in the following list are true?

1. At least one odd-numbered statement in this list is false.
2. Either the second or third statement in this list is true.
3. This list does not contain two consecutive false statements.
4. There are at least two false statements in this list.
5. If the first statement in this list is deleted, the number of true statements will decrease.

Q.940 Write a polynomial $p(x)$ in the following form:

$$p(x) = a_0 + b_0x + c_0x^2 + d_0x^3 + a_1x^4 + b_1x^5 + c_1x^6 + d_1x^7 + a_2x^8 + \cdots,$$

where all the coefficients are real numbers. Show that $p(x)$ is divisible by $x^2 + 1$ if and only if

$$a_0 + a_1 + \cdots = c_0 + c_1 + \cdots \quad \text{and} \quad b_0 + b_1 + \cdots = d_0 + d_1 + \cdots.$$