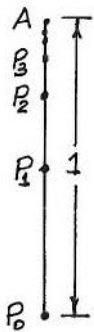


THE POWER OF PURE THOUGHT

The law of motion for a freely falling body was first enunciated by Galileo in the 17th century. When Galileo was contemplating what this might be he made the surprising discovery that one of the most natural, seemingly plausible laws of motion was not possible in any universe! Galileo discovered that the speed of a freely falling body cannot be proportional to the distance travelled. That is, in modern notation, it cannot be the case that $v \propto s$ or that $v = ks$ where k is a constant. To see why not let's measure time in seconds and suppose that $k = 1$ by measuring length accordingly.



Galileo considered the situation where the body falls from rest, from point A on the diagram, to a point P_0 , one unit of length below A . Let P_1 be the point halfway between A and P_0 , P_2 be the point halfway between A and P_1 and, generally, let P_n be the point halfway between A and P_{n-1} . Suppose t_n denotes the time taken to fall from P_n to P_{n-1} . Galileo's insight depends on the observation that, for each n , $t_n > \frac{1}{2}$ sec. Why is this? The length of $P_n P_{n-1}$ is $\frac{1}{2^n}$

and the maximum speed of the body over the interval $P_n P_{n-1}$ is $\frac{1}{2^{n-1}}$. (This speed occurs at the point P_{n-1}). Therefore $t_n > \text{distance}/\text{maximum speed} = \frac{1}{2^n} / \frac{1}{2^{n-1}} = \frac{1}{2}$. But the body must fall through each of the points P_n to reach P_0 . Therefore the time taken to fall from A to P_0 is at least

$$t_1 + t_2 + t_3 + \dots \geq \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \dots = \infty.$$

In other words it must take an infinite amount of time to fall from A to P_0 which means motion is impossible.