Parabola Volume 31, Issue 3 (1995)

MATHEMATICS AND THE HUMAN CONDITION

Derek W. Robinson[1]

A professor of mathematics at the University of Queensland recently recounted that when she telephoned Griffith University some years ago and asked for the Mathematics Department the receptionist replied, 'We do not have a Mathematics Department. Mathematics is not relevant to the Human Condition." This attitude is no great surprise since the depth, beauty and relevance of mathematics is rarely appreciated, nor its universal nature. The universality of mathematics arising from disparate cultures, arabian and egyptian, indian and mayan, chinese and greek, is in striking contrast to the diversity appearing in other cultural characteristics such as language, religion or music. Mathematics has thrived and developed because of its universal strength in modeling and predicting physical and biological phenomena and providing a reliable, replicable, framework for technological disciplines such as engineering or for social disciplines as economics. Mathematics provided the basis for early commerce, the possibility for safe navigation and the means to develop world trade. It has been unreasonably effective and efficient in an amazing number of areas. The aesthetic and cultural value of mathematics, its effectiveness in the structural analysis of a wide variety of phenomena and its power for predicting new phenomena are the reasons that mathematics is relevant to the human condition.

Society through its lack of appreciation of mathematical values often has difficulty according a suitable balance to mathematics' two dominant features, the aesthetic intellectual and the applied technological. Currently government and university policy place an overemphasis on the utilitarian aspects as opposed to the abstract ones. But in practice these are often inextricably intertwined. This can be illustrated by some examples from the spectrum of activities of the Centre of Mathematics and its Applications (CMA).

The first example, *mean-curvature flow,* provides a description of complex physical effects through simple geometric concepts. Imagine a globule of viscous fluid, motor oil, molasses or molten plastic. Left in isolation it adopts a spherical shape; its surface adapts to have the minimum curvature commensurate with its integral globular form. This property, the tendency of the surface to flatten as much as possible, is independent of the choice of viscous substance. The evolution of the surface is a direct function of its curvature. This observation provides a simple conceptual understanding of a wide range of flow problems and is of fundamental importance since

[1] This article first appeared in the **ANU Reporter**. Derek Robinson is Professor of Mathematics at the Australian National University where he is the Director of the Centre of Mathematical Analysis (CMA). Professor Robinson is the current President of the Australian Mathematical Society. The Editor thought that the contents of this article would be interesting to readers.

it allows a method of quantifying and predicting the flows. This has immediate relevance to the modern industry of injection molding.

The bumpers and dashboards of modern cars are typically constructed by manufacturing a mold of suitable proportions with a variety of injection points. The molten plastic is injected at each point. If the mold is correctly designed, with optimal placing of the injection points, the plastic fills the mold in a homogeneous manner. Everything depends upon the design of the mold. Since the construction of prototypes is an expensive business it is important to get it right. Hence it is essential to have a good understanding of the basic phenomena at work. A Japanese motor company under pressure to produce a new model tackled the design problem empirically by constructing five molds as variations of previous designs and then discarding the less satisfactory at a total cost of four million dollars. Thus a little inexpensive mathematics can lead to large rewards. Research on mean-curvature flow carried out at the CMA is now being exploited by a Melbourne company in the design of bumper molds. This application was certainly not foreseen by the original researchers but is an immediate development of basic research.

The second example is a project undertaken in collaboration with the CSIRO aimed at developing new techniques for characterising surface roughness. This project uses *fractal methods* which stem from work carried out in the old pure mathematics department of ANU in the 1970s, work which has had a broad international impact. Traditional approaches to the problem of surface roughness, for example those based on measures of the 'average fluctuation' of surface height, are very scale-dependent. In particular, the values of these fluctuations, usually recorded on a microscopic scale, depend on whether surface height is measured in metric or imperial units. In contrast, the fractal dimension of a surface is scale invariant –fractal analysis provides a way of dividing surface roughness into scale-free and scale-dependent components. The smoothest surfaces have fractal dimension two, as one would expect, but rougher surfaces have fractal dimension between two and three, the value increasing as the surface becomes rougher. Thus, fractal dimension is an index of roughness. It may be calculated empirically from surface data.

The problem of surface analysis arose initially in a consulting project with an Australian sheet metal manufacturer interested in monitoring the quality of its product by quantifying the roughness of metal sheets. The CMA and CSIRO mathematical scientists studied existing approaches to estimating fractal properties and found serious deficiencies. In the worst case encountered an oft-used method was shown to produce heavily biased results. Alternative substantially improved methods were developed and applied to a variety of new practical settings as well as to the original sheet metal problem. One application was to the development of an advanced, smooth and semi-permeable plastic film in which perishable goods are wrapped for storage and export. The film allows oxygen to pass in and carbon dioxide to pass out, but does not permit gas transfer in the opposite directions. All other things being equal, the film should be constructed with as smooth a surface as possible, offering least opportunity for microorganisms to adhere and promote deterioration of the goods.

The third example involves *wavelets.* Wave theory has had a long history of application to a wide range of subjects from acoustic to quantum mechanics. The relevant phenomena are modelled with superpositions of regular waves with different amplitudes. It was a realization of Heisenberg, expressed by his famous uncertainty principle, that this approach is incapable of giving accurate simultaneous estimates of position and velocity. The wavelet method which evolved in the 1970s overcomes this problem and consequently can be used to develop unusually accurate approximations to particularly complex phenomena, approximations which require a relatively low level of computational labour. These features have made wavelet methods exceptionally valuable in the study of computer vision, speech recognition and fractal analysis, to name only a few applications. They facilitate the recovery of complex signals from noisy data, a problem which has been of interest to scientists for more than half a century and which is of fundamental significance in an era of electronic communication. In 1995 the CMA will host a special year of wavelet theory and many of the world's leading experts have indicated their intention to visit Canberra during the year.

All three examples of the CMA research involve mathematical concepts, mean-curvature flow, fractals and wavelets, which have evolved in the last twenty years. These examples are striking since each concept was developed independently of any specific application but each is now of major importance in a wide variety of commercial and industrial applications. These examples illustrate the shortcomings of the 'project oriented', 'invent a better mousetrap', philosophy endemic in current Australian research management. Northcote Parkinson recounted satirically the tale of a management committee that took two minutes to dispense two million dollars on a reactor but two hours to spend two hundred dollars on a bicycle shed, they all understood the latter proposal. A contemporary committee would in contrast spend two hours on the reactor and two minutes on the bicycle shed, and then award two hundred dollars for the 'speculative', 'experimental', reactor and two million dollars for the 'well-conceived', 'achievable', bicycle shed.