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PROBLEM SECTION

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year or form. Solutions of these problems will appear in the next issue of **Parabola**; your solution(s) may be used if they are received in time.

Q.985 For what values of the positive integer n is

$$\text{(a)} \quad 5n+2 \qquad\qquad \text{(b)} \quad 7n+2$$

a perfect square? **ANS.** The following table lists the remainders $x^2(\mod 5)$ and $x^2(\mod 7)$ when the squares of the numbers $x = 0, 1, \cdots 6$ are divided by 5 and 7:

$x=0$	1	2	3	4	5	6	7	8	$\cdots$
$x^2=0$	1	4	9	16	25	36	49	64	$\cdots$
$x^2(\mod 5)=0$	1	4	4	1	0	1	4	4	$\cdots$
$x^2(\mod 7)=0$	1	4	2	2	4	1	0	1	$\cdots$

where the above pattern is repeated over and over.

(a) From the above table, there are NO squares of the form $5n+2$.

(b) Again from the above table, x^2 is the form $7n+2$ whenever x is of the form $7r+3$ or $7r+4$, i.e.

$$\begin{array}{lcrcr} x & = & 74+3 & \text{or} & 74+4, \text{ i.e.} \\ 7n+2=x^2 & = & 49r^2+42r+9 & rmor & 49r^2+56r+16 \\ 7n & = & 49r^2+42+7 & \text{or} & 49r_5^2 6r+14 \\ n & = & 7r^2+6r+1 & \text{or} & 7r^2+8r+2 \end{array}$$

where r is an integer.

Q.986 If f is a function, then the notation $f^2(x)$ means $f(f(x))$ and, in general, $f^n(x)$ means $f(\cdots f(x)\cdots)$ where there are n f's. If f is the function

$$f(x) = \frac{x-1}{x+1},$$

find $f^{1000}(3/4)$.

Q.987 You are in the process of finding the midpoints of the sides of a triangle using a ruler (with no measurements marked on it) and a pair of compasses. However, just

as you have constructed one of the midpoints P, you lose your compasses. Fortunately you notice that, if you place your ruler (which has parallel edges) along one of the sides of the triangle, then the point P lies on the opposite edge of the ruler. How would you construct the other two midpoints?

Q.988 Find all positive integers r, s, t such that r, s, t have no factor in common and

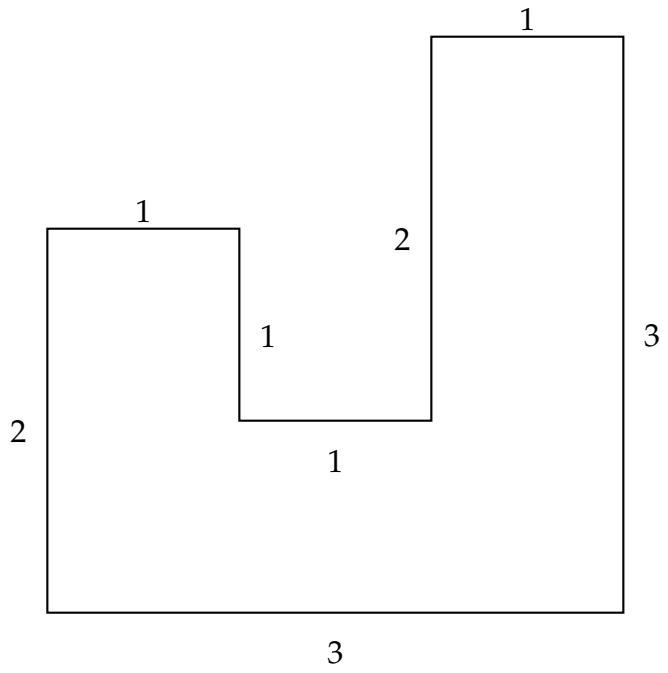
$$\begin{array}{lll} r & \text{divides} & s+t \\ s & \text{divides} & t+r \\ t & \text{divides} & r+s. \end{array}$$

Q.989 Suppose a is a real number between 0 and 1. Find all numbers x such that

$$[x] = ax$$

where $[x]$ denotes the greatest integer less than or equal to x (see problem 975).

Q.990 Find the area of the largest 6-sided figure (not necessarily rectangular) which can be drawn inside the 8-sided figure shown.



Q.991 The letters a, b, c, d, e, f represent the numbers $1, 2, 3, 4, 5, 6$ in some order. It is know that

$$\begin{array}{lll} & a+b & < & c+d \\ \text{and} & c+e & < & a < f. \end{array}$$

Find the values of a, b, c, d, e, f (in order).

Q.992 In a given (co-ed) school, each boy has gone out with at least one girl (but not every girl) and each girl has gone out with at least one boy (but not every boy). Show that there are two boys B, and B_2 and two girls G_1 and G_2 such that each of B_1 and B_2 has gone out with exactly one of G_1 and G_2, and each of G_1 and G_2 has gone out with exactly on of B_1 and B_2.