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PROBLEM SECTION

Q.1001 On the island described in question 2 of the Junior Division for this year's mathematics competition, each town entered one Australian Rules football team in the national championship. The teams are nicknamed the Bears, Cats, Swans, Magpies, Tigers and Hawks – just a meaningless coincidence, of course. One game was played between any two teams, with the winner scoring four points and the loser none, or two each in case of a draw. The eventual winners were the Magpies, with a score of 18 points. The only team they did not beat was the Hawks, who scored just 4 points; in fact the Hawks were the only team not to win a game. The runners-up were the Cats with 12 points, followed by the Swans and Tigers with 10 points each; these three teams lost just one game each. Given that the Tigers did not beat the Bears, find the results of all fifteen games.

By another meaningless coincidence, two teams drew their match if and only if their home towns are connected directly by a road. Which team came from which town?

Q.1002 Extend problem 4 in the Junior Division of the competition to the case where $\triangle ABC$ is isosceles. Find the ratio

$$\text{area }(\triangle A'B'C')/\text{area }(\triangle ABC)$$

in terms of the base angles of $\triangle ABC.$

Q.1003

(a) In how many ways can we choose n integers $x_1, x_2, \ldots, x_n$ such that each is $0, 1$ or 2 and their sum is even?

(b) What if the numbers are to be $1, 2$ or 3 and their sum even?

Q.1004 Is it possible to circumscribe two non-congruent triangles of equal area around the same circle?

Q.1005 Prove that if p and q are odd numbers then $p^2 - 4q$ is not a square.

Q.1006 Show that 343 is a factor of $2^{147} - 1.$