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PROBLEM SECTION

David Angell has handed over responsibility for the problem section to Peter Donovan. David retains principal responsibility for the Schools Mathematics Competition.

This set of problems is different from usual. It is almost 'classical', that is made up of quite old problems. The oldest, number 1017, goes back to Archimedes (290–212 BC)! The last problem is on physics rather than mathematics.

Q.1015 Quantities of coins are available denominated at one tenth, one twelveth and one sixteenth of a penny. How can these be used to settle a debt of one two hundred and fortieth of a penny? The giving of change is allowed.

[At various times in various parts of the British Empire various low value coins were made available to supplement the standard imperial coinage. Coins of the above denominations were used in Nigeria, Malta and Ceylon respectively.]

Q. 1016 Show how six cylindrical pencils of equal radius each with neither end sharpened can be put into mutual contact along their curved surfaces.

[This problem was set by Martin Gardiner in his column in *Scientific American*. To his surprise someone worked out how to put seven such pencils into mutual contact. Do this too.]

Q. 1017 By considering an equilateral triangle inscribed in a unit circle show that $2\pi > 6\sin 60°$. Next consider a regular 6-gon inscribed in the same circle and show that $2\pi > 12\sin 30° > 6\sin 60°$. Thus show that $2\pi > 192\sin 1.875°$.

Now, using only the square root facility on your pocket calculator and the formula $\sin\theta = \sqrt{\dfrac{(1-\sqrt{1-\sin^2 2\theta})}{2}}$ which is valid for $0° \le \theta \le 180°$, to find (decimal) fractions r_1, r_2, r_3, r_4, r_5 such that sin $30° > r_1, \sin 15° > r_2, \sin 7.5° > r_3, \sin 3.75 > r_4, \sin 1.875° > r_5$ and $96r_5 > 3\frac{10}{71}$. You have now worked out the substance of Archimedes' proof that $\pi > 3\frac{10}{71}$.

Now prove that $3\frac{1}{7} > \pi$ in this style.

Q.1018 You are given two 20-litre containers full of milk, one empty 5-litre container and one empty 4-litre container. No other measuring container is available. How do you pour milk from one container to another so as to end up with exactly two litres in each small container, 20 litres in one large container and 16 litres in the other?

[This is due to Henry Dudeney (1857–1930) and is problem 410 in Martin Gardiner's edition of *536 Puzzles and Curious Problems.*]

Q.1019 How can a regular hexagon be dissected by straight line cuts into five pieces which can then be re-assembled to make a square?

[This is problem 311 in Gardiner's edition of Dudeney's problems. Much more on this sort of thing is in Harry Lindgren's *Recreational Problems in Geometric Dissections* (Dover).]

Q.1020 Express the fractions $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}$ and $\frac{1}{9}$ each as the quotient of a 4-digit number by a 5-digit number in which each of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 occurs in either the numerator or the denominator.

[This is due to Sam Loyd.]

Q.1021 When Mr. Banquo, Mr. Macbeth, Mr. Macduff, Mr. Duncan and Mr. Donalbain all accepted parts in an amateur production of *Macbeth*, complications became inevitable. They were heightened by the stage director's decision to give each of these gentlemen the role of which another one is the namesake. Nor was this all. At the last moment, the stage director decided to change all the roles round; and, when the play was finally put on, none of our Thespians played either the role of which he was the namesake, or the part he has rehearsed.

For example, Macbeth was played by the namesake of the part rehearsed by Mr. Banquo. Macduff was played by the namesake of the part rehearsed by Mr. Donalbain. Mr. Macduff, who had rehearsed the part of Donalbain, played that of Banquo.

Who played the part rehearsed by Mr. Macbeth?

[This is problem 41 in the collection of 100 problems by Hubert Phillips published by Dover in 1961. Such problems used to be published in various newspapers and magazines in Britain and America.]

Q.1022 Are there more ways of paying \$1 using coins of denominations 1c, 2c, 5c, 10c, 20c and 50c than there are of paying \$5 using coins of denominations 5c, 10c, 20c, 50c, \$1 and \$2?

Find the precise number of ways in each case.

Q.1023 Given 12 apparently identical objects of which 11 have the same weight and 1 has a slightly different weight, show that it is possible to use a simple balance three times to determine which one has the different weight and to determine whether its weight is greater than the weight of any of the others.

Now suppose that one has 13 apparently identical objects of which 12 have the same weight and 1 has a slightly different weight. Show that there is no such procedure to determine which one has the different weight and whether its weight is greater than the weight of any of the others.

Finally, let N be such that given N apparently identical objects of which $N-1$ have the same weight and 1 has a slightly different weight it is possible to use a simple balance four times to determine which one has the different weight and to determine whether its weight is greater than the weight of any of the others. What can be said about N?

Q.1024 A frictionless pulley is affixed from the ceiling and a weightless rope passed through it. [Yes, all this is an idealisation.] An inert object is attached to one end of the rope. On the other side of the pulley a monkey of the same weight holds on to the rope. If the monkey climbs up the rope, what happens to the inert object?