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HOW TO CONSTRUCT REGULAR 7-SIDED POLYGONS – AND MUCH ELSE BESIDES

Intended Results of the Paper-Folding Experiments Posed at the end of Part 1

by

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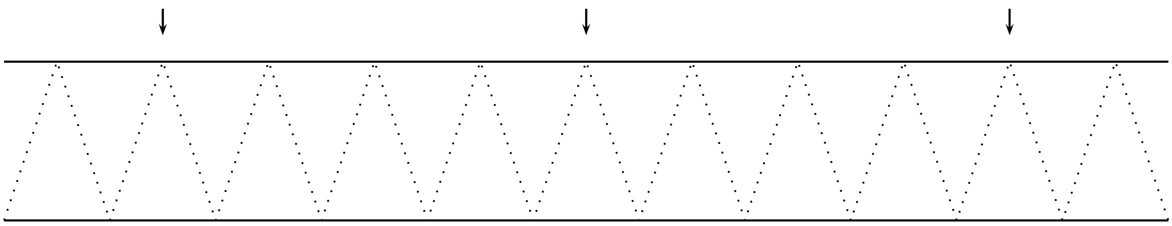
Experiment 1: Figure 1(a) shows a portion of tape which has been folded using the D^1U^1-procedure, with the first few (say 10) triangles cut away. By executing the FAT algorithm at the points located next to the symbol ↓ you may obtain the FAT triangle illustrated in Figure 1(b). The shading on the triangles at the vertices here, and on other parts of Figures 2 and 3, simply indicates that it is the under side of the tape that is visible there. The FAT triangle may be finished by gluing the tape together where it overlaps on a side of the triangle. We observe that the smallest angle produced on the D^1U^1-tape is $\frac{\pi}{3}$.



(a)

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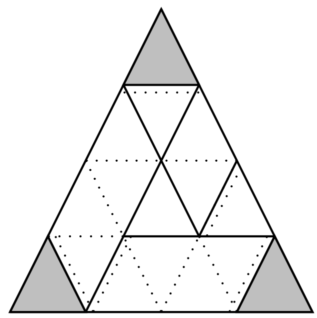
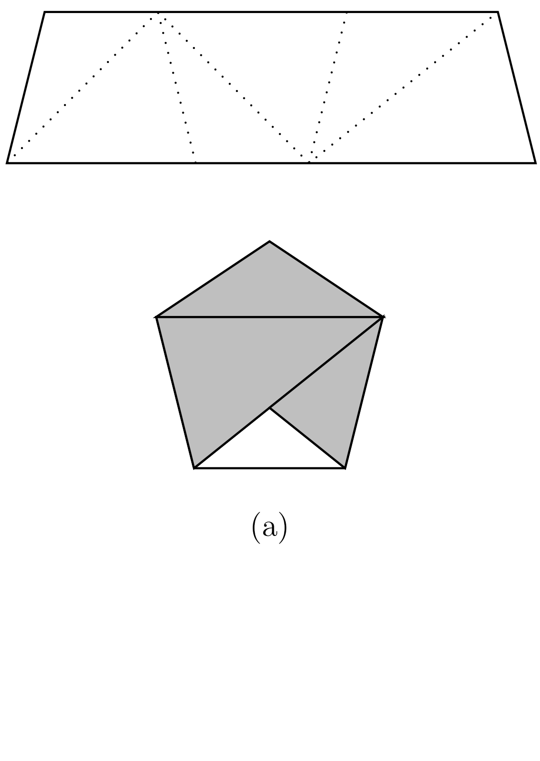


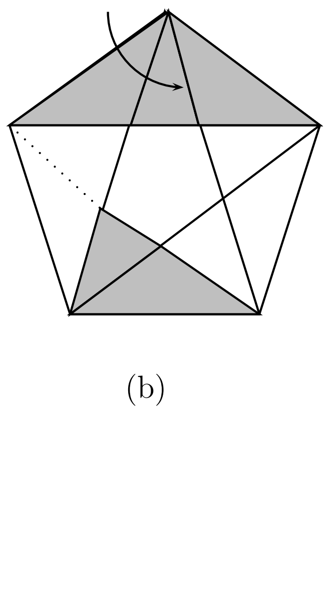
(b)
Figure 1

Experiment 2: Figure 2(a) shows a short portion of tape which has been folded using the D^2U^2-procedure, with the first few (again, 10 will do) triangles away. Notice that this tape has short and long lines on it. If the strip of 6 triangles (where the cuts were made on short lines) in Figure 2(a) is folded on only the short crease lines you get what we call the 'short-line' pentagon shown there. Figure 2(b) illustrates what happens if you take a longer strip of this tape and fold it only on long crease lines. The resulting figure is what we call the 'long-line' pentagon. Figure 2(c) illustrates the FAT pentagon which is obtained by executing the FAT algorithm at every other vertex along a much longer strip of the tape. We observe that the smallest angle produced on the D^2U^2-tape is $\frac{\pi}{5}$.

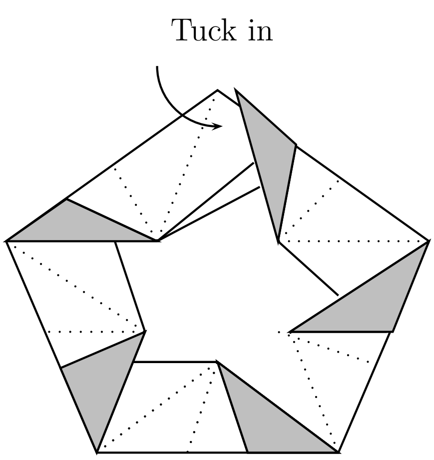
It is not worthwhile worrying about how many triangles are needed for any of these constructions, since once you have the strip of tape in hand it is easy to simply experiment until you get what you want and then cut away the excess.



(a)



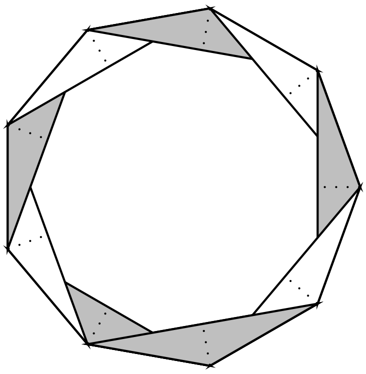
(b)



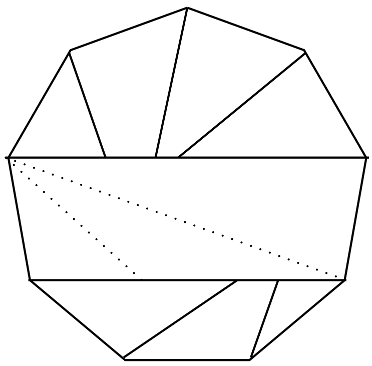
(c)

Figure 2

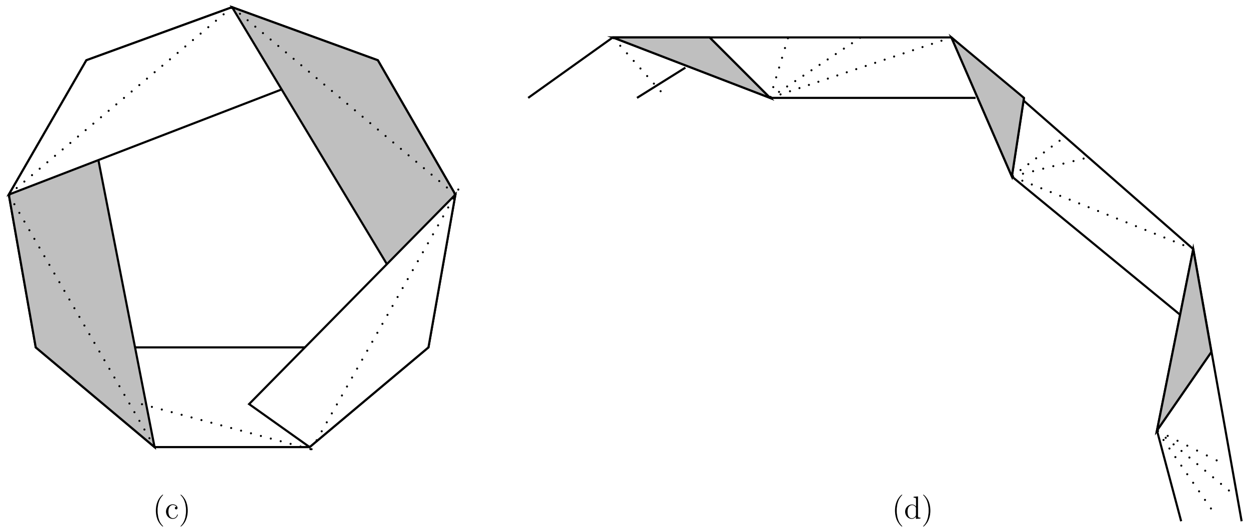
Experiment 3: The most popular wrong guess as to what will happen when you fold using the D^3U^3-procedure is that you will get tape that produces regular 7-gons. We think you might not have fallen into this trap if you remembered that the D^2U^1-procedure produced regular 7- gons. Still, strange things do happen in mathematics. But not that strange as you will see from the analysis in Part 2. Actually what you get from the D^3U^3-procedure is tape on which the smallest angle is $\frac{\pi}{9}$ and (as usual, throwing away the first 10, or so triangles) it may be used to produce regular 'long-line', 'medium-line', 'short-line' and FAT 9-gons as illustrated in Figures 3(a), (b), (c), and (d), respectively.



(a)



(b)



(c) (d)

Figure 3

A systematic answer to what you obtain by adopting the general period-1 folding procedure, that is, by using the D^nU^n folding procedure, is given in the second part of our article.