

## PROBLEMS SECTION

Correspondence to [p.donovan@unsw.edu.au](mailto:p.donovan@unsw.edu.au)

**Q.1025** Find the smallest number that when divided by 29 leaves the remainder 23 and that when divided by 37 leaves the remainder 31.

**Q.1026** Find the smallest number that when divided by 29 leaves the remainder 23, that when divided by 37 leaves the remainder 31 and that when divided by 43 leaves the remainder 41.

**Q.1027** Find the maximum of  $3x + 4y$  where  $x, y$  are subject to  $x^2 + y^2 = 1$ .

Interpret your answer geometrically.

Now find the maximum of  $8x + 4y + 1$  where  $x, y, z$  are subject to  $x^2 + y^2 + z^2 = 1$ .

**Q.1028** If  $\binom{n}{k}$  denotes the binomial co-efficient, show that  $\sum_{k=0}^n k \binom{n}{k} = 2^{n-1}$ .

**Q.1029** Show that  $\sqrt[3]{7 + 5\sqrt{2}} + \sqrt[3]{7 - 5\sqrt{2}} = 2$ .

**Q.1030** A citizen is sitting in a rowing boat floating in a small lake. There is a large steel anchor in the boat. If the citizen throws the anchor overboard does the level of the lake rise or fall? Does the level of the boat relative to the shore rise or fall?

**Q.1031** Three circles each of radius  $a$  are in mutual contact. Find the radius of the circle that circumscribes all three.

**Q.1032** Four spheres each of radius  $a$  are in mutual contact. Find the radius of the sphere that circumscribes all four.

**Q.1033** What rate of interest paid in advance on a 1-year loan is equivalent to an interest rate of  $r\%$  paid at the end of the term of the loan?

**Q.1034** Suppose that the diagram shown has been drawn in pencil. It is desired to draw ink squares over the pencil lines so as to cover the entire design with ink. What is the

minimal number of ink squares needed to carry out this task?

