

PROBLEM SECTION

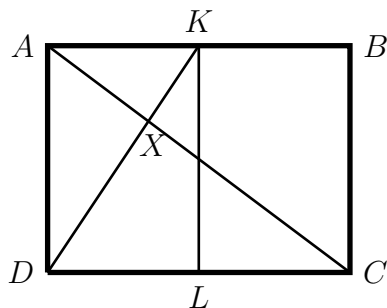
You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year or form. Solutions of these problems will appear in the next issue of **Parabola**; your solution(s) may be used if they are received in time.

Q.1035 Find all positive integers n and m such that n is a factor of $4m - 1$ and m is a factor of $4n - 1$.

Q.1036 Prove (without using induction) that $2^{4n} - 15n - 1$ is divisible by 225, for all positive integers n .

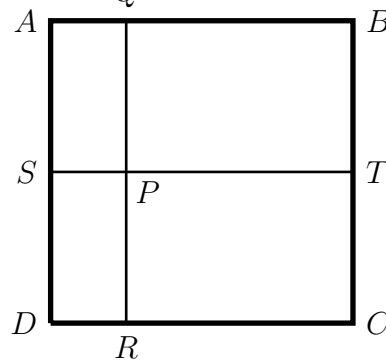
Q.1037 Suppose n is a positive integer and let $f(x) = \frac{x}{1+n^2x^2}$. Without using calculus, show that $f(x) \leq \frac{1}{2n}$. (Hint: Look at $\frac{1}{f(x)}$.)

Q.1038 Let $ABCD$ be a rectangle, and K, L be the midpoints of AB and CD respectively. Suppose AC and KD meet at X . Find the ratio of the area of $\triangle KXA$ to the rectangle $ABCD$.



Q.1039 Let M, N be positive integers. If $x^M(1-x)^N$ is divided by $1+x^2$ giving a remainder of $ax + b$ show that $a = (\sqrt{2})^N \sin\left(\left(2M - N\right)\frac{\pi}{4}\right)$ and $b = (\sqrt{2})^N \cos\left(\left(2M - N\right)\frac{\pi}{4}\right)$. (Hint: Complex numbers may be useful.)

Q.1040 Let $ABCD$ be a unit square and let ST be any line passing through the square which is parallel to AB and QR be a line passing through the square which is parallel to the side BC . Suppose that these lines meet at P . Show that if the rectangle $QPTB$ has area larger than $\frac{1}{4}$ then the rectangle $DSPR$ has area smaller than $\frac{1}{4}$.



Q.1041 Suppose we have $n + 1$ positive integers all less or equal to $2n$. Prove that among this list there must be an integer that divides one of the other integers.

Q.1042 The famous sleuth Hercule Poirot has discovered the following facts.

- Professor Edelstein was not in the study at the time of the murder.
- Either Lady Adeline committed the murder or Sir Benjamin did.
- If Countess Delilah was not in the study then Professor Edelstein was in the study.
- If Sir Benjamin committed the murder then Cecil was in the dining room at the time of the murder and Countess Delilah was not in the study.

Who committed the murder?