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LENGTH AND RELATIVITY

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Time Measurement and Relativity

In a previous issue issue of **Parabola** (Vol 29 No 2 p.2), I discussed the effect on time measurement of Einstein's two postulates of Special Relativity. These two postulates are:

1. *the laws of Physics are the same to any inertial observer* and
2. *there is an inertial observer for whom light signals in vacuum travel at a constant speed in all directions whatever the motion of the light source*.

An **inertial observer** is one for whom Newton's First Law holds: *an object on which no force acts moves in a straight line at constant speed*. The speed of light is represented by the symbol c and is about 2.988×10^8 metres per second.

Combining Einstein's two postulates (in the previous article) leads us to the conclusion that, in vacuum, light travels at speed c in all directions at all times according to all inertial observers (however fast they or the light source are going).

Armed with these two postulates and as much clear thinking as we could muster, we looked closely at time measurement. We found that a moving clock will run slowly by a factor called the **gamma factor**, where for an object moving at speed v

$$\gamma = (1 - v^2/c^2)^{-1/2}. \tag{1}$$

We also dealt with the order of distant events in that article, finding that two events occurring in different places that are simultaneous to one observer are not necessarily simultaneous to another, and that the order in which two events occur can also depend on the observer. In this article we consider how differences in the length measurements also depend on the gamma factor.

Measuring Length

Suppose we are an inertial observer, and are watching a rocket moving at speed v metres per second towards us from a distance L metres away. Since distance is speed $\times$ time, the rocket will reach us after L/v seconds. However, time dilation means that the clocks on the rocket are running slowly, and so if they were reading 0 when the rocket was L metres away, they are reading $L/(v\gamma)$ when they reach us.

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Now the same thing must be observed by anyone on the rocket: from their point of view we have approached them at speed v, and have taken $L/(v\gamma)$ seconds to reach them. It follows that, if relativity is to be consistent, that from their point of view, we did not start L metres away, but were at a distance of L/γ. In other words, when moving, distances are *shortened* by a factor of γ.

Just as in the case of time dilation, this effect is not just an accident of our way of measuring, it is a real effect. Unfortunately, no experimental verification of this **length contraction** has actually been done yet. However we see from the above argument that time dilation and length contraction must both occur if one does, or we are led to contradictions, and time dilation has been measured.[2]

For historical reasons, length contraction is sometimes called **FitzGerald-Lorentz contraction**, and James Coleman in *Relativity for the Layman* quotes the limerick:

There was a young fellow called Fisk
Whose fencing was exceedingly brisk;
So fast was his action
The FitzGerald contraction
Turned his rapier into a disc.

Now that we have the two ideas of time dilation and length contraction, we can explain how it is (in theory) possible to travel to a distant star, say 200 light years away, within a human lifetime. Recall that a light year is the distance that light travels in one year, about 6 million million miles (or $9\frac{1}{2}$ million million kilometres). As nothing can travel faster than light, how could we hope to get to such a star from earth within one lifetime?

Both time dilation and length contraction supply the answer. Suppose that a plucky astronaut makes the trip at a speed with $\gamma = 4$, that is at approximately 96.8% of the speed of light (about 289 thousand kilometres a second). From the point of view of the earth (which we take as inertial for the sake of argument), the astronaut takes 206.5 years to make the trip. However, time dilation means that the astronaut only experiences a quarter of this time span, or 51.64 years.

On the other hand, as far as the astronaut is concerned his clocks are running perfectly (the clocks on earth seem slow though), but the distance he has to travel is not 200 light years, but only 50. At 96.8% of the speed of light such a trip will take 51.64 years.

We see that if distances did not contract due to motion, we would be left with no way to resolve the conflict between the time the astronaut measures he takes for his trip and the fact that nothing can travel faster then light. If the distance to the star were not shrunk for the astronaut, then he would have travelled 200 light years in just over fifty of his years, that is at nearly four times the speed of light (as he measures it). This is not possible in relativity.

The Pole in the Barn Paradox

[2]See, for example, "Around-the-World Atomic Clocks: Predicted Relativistic Gains" J.C. Hafele and R.E. Keating, Nature Vol. 177 (1972) 166-170.

Now we know about length contraction, we can invent some amusing uses of it.

Suppose you want to fit a 20m pole into a 10m barn. If the pole were moving fast enough, then length contraction means it would be short enough. For the figures we have here, we need $\gamma = 2$, and that works out at a speed of about 86.6% of the speed of light, or 259.8 million metres per second.

Now comes the paradox. According to your friend who is going to slam the barn doors shut just as the end of the pole goes in, the pole is 10m long, and therefore it fits. However as far as you are concerned, the pole is still 20m long but the barn is now only 5m long: length contraction must work both ways by the first postulate. How can you fit this 20m pole into a 5m barn? This paradox is apparently due to Wolfgang Rindler of the University of Texas at Dallas.

Of course the key to this is relativity of simultaneity. Your friend sees the front end of the pole hit the back wall of the barn at the same time as the doors are closed, but you (and the pole) do not see things this way. You are standing still and see a 5m long barn coming towards you at some shockingly high speed. When the back of the barn hits the front of the pole (and takes the front of the pole with it), the back end of the pole must still be at rest. It cannot 'know' about the crash at the front, because the shock wave travelling along the pole "telling it" about the crash travels at some finite speed. The front of the barn has only 15m to go to get to the back of the pole, but the shock wave has to travel the whole length of the pole, namely 20m. The speed of the barn is such that even if this shock wave travelled at the speed of light, it would not get to the back of the pole before the front of the barn did. Hence in both frames of reference, the pole fits inside the barn (and will presumably shatter when the doors are closed).

An important point to take from this is that if we get one result from correctly reasoning as one observer, then the same result must be true to any other inertial observer: we may need different reasoning though.

The Magician's Assistant Paradox

We can use the same principle to resolve another paradox.

The Magician's Assistant Paradox is the following (which I first had outlined to me by John Pulham of the University of Aberdeen): A magician is performing a trick with two guillotines set 160cm apart. Her assistant (who is 2m tall) is sent towards the guillotines, while lying down on a trolley. This trolley is moving at a fraction over 60% of the speed of light, so γ is slightly greater than 1.25. The magician drops the blades so that they fall when the assistant is exactly between the guillotines, which will miss him as length contraction makes him slightly less than 160cm tall to the magician and the guillotines. The blades then drop out of the assistant's way and he continues on unharmed.

But from the point of view of the assistant, the two blades are rushing towards him at 60% of the speed of light and, rather than being 160cm apart, are less than 128cm apart, and are therefore likely to cut him into three pieces.

We can resolve this paradox by explaining exactly what the assistant sees when the blades fall. Once again, it is the relativity of simultaneity that saves the assistants neck: one blade must fall before the other. In fact, we can see that the blade furthest from him must fall first (just in front of the top of his head), so that he goes *over* this blade in both his frame and the magician's. Then just as his feet pass the second blade, that one falls, and he goes *under* this blade in both his frame and the magician's. If you do the exact calculations, you find that this is indeed what happens.

Conclusion

What we have been discussing in this article is the effect that motion can have on length and distance. The mathematics is simple, but the ideas behind what we are doing are subtle, and in places rather sophisticated. The key point is that when moving, distances are shrunk. As in the case of time distortion we discussed in the previous article, this effect is only apparent in comparison to another inertial observer, and is symmetric.

If you are be interested in relativity, there are many books on the subject: I would particularly recommend Wolfgang Rindler's *Introduction to Special Relativity*. For those with WWW access, there is a list of frequently asked questions — which includes a discussion of the Pole in the Barn paradox — kept at

`www.phys.unsw.edu.au/physoc/physics_faq/relativity.html`[3]

There are many web sites dealing with Special and General Relativity, but you should be careful of the large number of alternate theories (one for each alternate theorist), as all of those I've seen are not worth perusing.

[3]Editorial note, February 2014. This is a dead link.