

## PROBLEM SECTION

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year or form. Solutions of these problems will appear in the next issue of **Parabola**; your solution(s) may be used if they are received in time.

**Q.1051** What is the fractional derivative

$$\frac{d^{\frac{1}{2}}f}{dx^{\frac{1}{2}}}$$

of  $f(x) = 1/\sqrt{x}$  (see the article on fractional calculus in this issue of Parabola).

**Q.1052** In a class of thirty students what is the probability that two or more students have their birthdays on the same day?

**Q.1053** A continued fraction is an expression of the form

$$[n_1, n_2, n_3, \dots] = n_0 + \frac{1}{n_1 + \frac{1}{n_2 + \frac{1}{n_3 + \dots}}}$$

A useful feature of continued fractions is that they can be used to provide a set of integers to approximate an irrational number. As an example consider

$$\sqrt{14} = 3.7416573867739413856\dots$$

This can be written as the continued fraction

$$[3, 1, 2, 1, 6, 1, 2, 1, 6, 1, 2, 6, 1, \dots] = [3, \overline{1, 2, 1, 6}]$$

where the  $\overline{1, 2, 1, 6}$  means that the numbers 1, 2, 1, 6 are repeated indefinitely. You might like to check the accuracy of the continued fraction approximations

$$\begin{aligned} [3, 1] &= 3 + \frac{1}{1} = 4 \\ [3, 1, 2] &= 3 + \frac{1}{1 + \frac{1}{2}} = 3.\overline{6} \\ [3, 1, 2, 1] &= 3 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1}}} = 3.75 \\ [3, 1, 2, 1, 6] &= 3 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{6}}}} = 3.\overline{740} \end{aligned}$$

What is the continued fraction for the Golden Section  $\phi = (1 + \sqrt{5})/2$ ?

**Q. 1054** What is the value of the infinite square root

$$\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}}}}$$

**Q. 1055** Let  $ABC$  be a triangle with  $A_1, B_1, C_1$  the midpoints of the sides  $BC, CA, AB$  respectively.

Let  $D$  be the foot of the perpendicular from  $A$  to  $BC$ .

Show that the triangles  $B_1C_1D$  and  $B_1C_1A_1$  are congruent.

**Q. 1056** Alice and Bob arrived at the dairy with their jugs, each asking for 2 litres of milk. The assistant serving them found that he had nothing to measure out the milk: all he had was an 80-litre can full of milk. Alice informed him that her jug held exactly 5 litres and Bob said that his held 4 litres.

- Find out how the assistant was able to measure out the 2 litres that Alice wanted into her jar.
- After Alice left, the assistant realised that there was another 80-litre can full of milk in the storeroom. How did he give Bob the 2 litres that he wanted?