PROBLEM SECTION

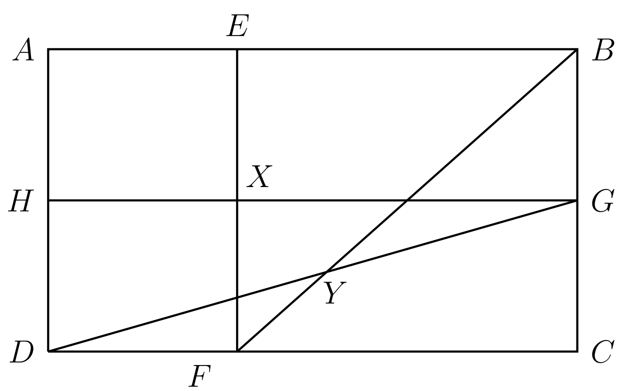
You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year or form. Solutions of these problems will appear in the next issue of **Parabola**; your solution(s) may be used if they are received in time.

Q1057 Remember that a regular polygon has all sides equal and all angles equal.

(a) Show that the two constructions of the regular pentagon at the end of Peter Merrotsy's article actually work.

(b) Starting with a regular pentagon, straight edge, and compass, construct a regular fifteen-sided polygon.

Q1058 Let ABCD be a rectangle. Let EF be a line segment parallel to AD and BC which divides the rectangle ABCD into two smaller rectangles AEFD and EBCF, and let GH be a line segment parallel to AB and DC which similarly divides ABCD into ABGH and HGCD. Let X denote the intersection of EF and GH, and Y denote the intersection of BF and GD. Show that A, X, and Y are collinear.



Q1059 Prove that there are infinitely many positive integer solutions x,y,z to the equation

$$x^7 + y^8 = z^9.$$

Q1060 The colour of each side of a wooden cube is chosen randomly, and independently of all other sides, from one of the three colours red, green, and blue.

What is the probability that the cube has at least one pair of opposite faces which have the same colour?

Q1061 In a tennis tournament every player plays every other player exactly once. We say that player X has *directly beaten* player Y if X plays against Y and wins; we say that player X has *indirectly beaten* player Y if X has directly beaten a player Z who has directly beaten Y. If a player has beaten all other players in the tournament (either directly or indirectly), that player is awarded a prize.

A tennis player (let's call him Pete) enters the tournament and ends up being the only player in the tournament to receive a prize. Show that Pete must have directly beaten everyone else in the tournament. (In tennis it is not possible to draw).

Q1062 Suppose that x is a real number such that

$$y = (x + \sqrt{x^2 + 1})^{1/3} + (x - \sqrt{x^2 + 1})^{1/3}$$

is an integer. Show that x is also an integer.

Q1063 For every real number $0 \le x < 1$, let $f(x)$ be the sum of the first 1999 digits in the *binary* expansion of x. (For example, since the binary expansions of $1/2$ and $3/4$ are 0.1 and 0.11 respectively, $f(0) = 0$, $f(1/2) = 1$, $f(3/4) = 2$, etc.) Compute $\int_0^1 f(x)^2 \, dx$.