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Problems Section

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year. Solutions to these problems will appear in the next issue of *Parabola*; your solution(s) may be used if they are received in time.

Q1171 The first digit of a 6-digit number is 1. If the 1 is shifted to the other end, the new number is 3 times the original number. Find this number.

Q1172 Find a number that has 10 divisors such that the product of the divisors is 60466176.

Q1173 In a rectangle, the shorter side is 4 times the difference between the diagonal and the longer side. Find the ratio of the longer side to the shorter side.

Q1174 Generalise the inequality in Q1168 (Vol 40, No 2, 2004) to the case of n positive numbers a_1, a_2, …, a_n, and prove it.

Q1175 Recall that if m and n are two integers such that $0 \leq n \leq m$, then $\binom{m}{n} = \frac{m!}{n!(m-n)!}$. Find the sum

$$S = \binom{2005}{0} + 2\binom{2005}{1} + 3\binom{2005}{2} + \cdots + 2006\binom{2005}{2005}.$$

Q1176 Let P be a point on the side AB of an equilateral triangle ABC. Let P_1 be the foot of the perpendicular from P to BC, P_2 be the foot of the perpendicular from P_1 to AC, P_3 be the foot of the perpendicular from P_2 to AB, etc. Show that as n increases indefinitely, the triangle $P_nP_{n+1}P_{n+2}$ is tending to become equilateral.

Q1177 Adam and Brian watch the sun setting over the ocean on a calm day. Adam is 10m above the sea level and Brian is on a cliff top 30m above Adam. How long after Adam does Brian observe the instant of sunset? (Take the circumference of the earth to be 40,000km.)

Q1178 Find a point M in a triangle ABC satisfying $\angle MAB = \angle MBC = \angle MCA$.

Q1179 Prove that $(a+b)^2 \geq a^2(1-\epsilon) - b^2(1+1/\epsilon)$ for any $a, b \in \mathbb{R}$ and $\epsilon \in (0,1)$.

Q1180 The function defined by

$$\zeta(n,m) = \sum_{k=1}^{m} \frac{1}{k^n} = \frac{1}{1^n} + \frac{1}{2^n} + \cdots \frac{1}{m^n}$$

where n and m are integers, is an example of a special function in mathematics known as the incomplete Riemann function. Show using elementary operations that

$$\sum_{k=2}^{m} \zeta(n, k-1) = m\zeta(n, m) - \zeta(n-1, m).$$

This problem and its answer (to appear in the next issue) was suggested by N.P. Singh.