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Problems Section

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year. Solutions to these problems will appear in the next issue of *Parabola*, and if received in time your solution(s) may be used.

Q1191. Let ABC be a triangle with sides a, b, c in the usual way and let r be its circumradius.

(i) Show that $3r > \dfrac{a+b+c}{2}$.

(ii) Let P be any point within ABC, and let r_1 be the circumradius of ABP. Is it true that $r_1 < r$?

Q1192. Three circles with centres O_1, O_2, O_3 and of equal radii r, all pass through a point P. Let their second points of intersection be A, B, C. Show that O_1, O_2, O_3 is congruent to ABC. What is the circumradius of ABC?

Q1193. $ABCD$ is a trapezium with $AB \parallel DC$ and $AD = AB + DC$. Let M be a point on AD such that $AM = AB$.

(i) Prove that $\angle BMC$ is a right angle.

(ii) Let F be the midpoint of BC. Prove that $\angle AFD$ is also a right angle.

Q1194. In the triangle ABC, M is the midpoint of BC. Points X and AB and Y on AC are such that $XY \parallel BC$. Show that BY and CX intersect at a point P on AM.

Q1195. Two circles intersect at A and B and l is a variable line through A which intersects the circles at X and Y, respectively.

(i) Show that as l varies, all the triangles XBY will be similar to each other.

(ii) Find out how to draw l such that

(a) XY is as long as possible,
and

(b) A is the midpoint of XY.

Q1196. ABC is a triangle, m_a is the median from A to the side $BC = a$.

(i) Show that $m_a < \dfrac{1}{2}(b+c)$.

(ii) If p is the perimeter of ABC then

$$\frac{3}{4}p < m_a + m_b + m_c < p.$$

Q1197. Given a circle, centre 0, radius r, and a point P outside the circle, construct a line through P meeting the circle in the points A and B such that $PA = AB$.

Q1198. Given two intersecting lines ℓ and m and a point P not lying on either line, construct a straight line through P meeting ℓ in A and m in B such that P is the mid-point of AB.

Q1199. Given a $\triangle ABC$, construct a square $XYZU$ such that side XY lies along BC, while vertex Z is on AC, vertex U on AB.

Q1200. The Happy Ending Problem.

Given any five points in the plane with no three points lying on a straight line show that it is always possible to select four of the points as vertices for a convex quadrilateral. (Note that a quadrilateral is convex if any two points inside the quadrilateral can be connected by a straight line segment that does not fall outside the quadrilateral.)