Parabola Volume 42, Issue 1 (2006)

History of Mathematics:
A New Way to apply Mathematics?

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The story I want to tell in this issue concerns a development I first learned of a little over 30 years ago, but whose roots lie much deeper than that. It was in 1973 that I was first introduced to the mathematical development known as Catastrophe Theory. This was first mooted in the context of an attempt to describe the process of 'morphogenesis' – the development of form from an initially amorphous (formless) basis. The obvious example of this is the process studied in the science of Embryology, which charts the progress of an initially undifferentiated mass of cells into the specialized structures of the later embryo. As I was at that time researching the application of Mathematics to Biology, this interested me greatly, and I sought to master the new branch of mathematics involved.

It was the brainchild of an eminent French mathematician, Ren Thom (1923–2002). Thom won the Fields medal (widely regarded as the mathematical equivalent of the Nobel Prize) in 1958 for his powerful and influential results in the topology of higher dimensional spaces. In the late 1960s, he turned his attention to wider concerns, prompted by his proof of a remarkable theorem on the classification of functions.

Generally, when some system is subjected to a continuous (smooth) change in the value of some input parameter, the result is a continuous change in its state, as measured by some output. However, there are some exceptions. The simplest such case is that of a cubic function

$$y = x^3 - ax$$

which, when (the single parameter) $a > 0$, possesses a single minimum which is found where $x = \bar{x} = \sqrt{a/3}$. However, when $a < 0$, no such minimum exists. (In many important cases, systems adopt configurations based on the minima of various functions.) This simple example in fact underlies a great number of 'threshold phenomena' such as the 'change of state' involved when a gas condenses into a liquid. Such sudden changes in output came to be called 'catastrophes'.

Thom considered all possible cases in which 1, 2, 3 or 4 parameters were involved and found that there were only seven different possible situations: seven different catastrophes. The possible ramifications of this discovery occupied him for much of the following decade or so. He developed his theory in the context of the 'Serbelloni conferences', select gatherings of diverse but acknowledged experts brought together by the embryologist C H Waddington with the avowed purpose of developing a systematic theoretical (i.e. mathematical) Biology.

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In 1972, Thom published his influential book on the subject, shortly afterwards translated as *Structural Stability and Morphogenesis*. This was the work that introduced Catastrophe Theory to a wider public. I was one of the early reviewers of it, and my consequent correspondence with Thom led to me organizing his visit to Australia in 1976. My conversations with him at that time helped me to understand much better the wellsprings of his ideas on the status of Mathematical Biology.

His most developed account of those wellsprings was his contribution to the fourth and last Serbelloni conference. This paper was entitled 'Structuralism and biology'. It begins with the commonplace observation that the big success stories of Applied Mathematics – Physics and Astronomy – have not been replicated in the 'softer sciences' of Biology and the various social sciences.

Thom's view of this situation was that we tend to ask the wrong questions and so to misdirect our effort when we undertake research in these areas. Instead of trying to *quantify* our investigations, he thought, we would better succeed if we sought *qualitative* conclusions: the aim should be not so much to measure as to classify. The title of his paper comes from a paradigm that is widely regarded (particularly in continental Europe) as having provided a successful methodology in some of the social sciences, most notably Linguistics and Anthropology.

This is an approach known as 'Structuralism', and commonly regarded as deriving from the linguistic researches of Ferdinand de Saussure (1857–1913). According to Saussure, language is a *structure* whose component parts can only be understood as the *set of relations between them*. They have no intrinsic meaning in themselves; the concept named and the word we use to name it hold no necessary relation one to the other.

Even the boundaries between words can exhibit a certain arbitrariness. Our English language distinguishes the words 'bloom' and 'broom', and assigns them different meanings, but to Japanese speakers they sound the same. This is because English and Japanese employ different structures in the formation of their words. [In contrast, we are unable to differentiate the two different '-ch-' sounds which the Chinese distinguish from one another.]

Saussure is often described as having stressed the synchronic aspects of language (the structure of the language at a particular time) rather than the *diachronic* (the course of linguistic evolution over time).

His ideas were embraced and extended by the émigré Russian linguist Roman Jakobson (1896–1982). Among Jakobson's concerns was the area of 'linguistic typology', in which languages are grouped into families on the basis of their similarities of structure, rather than on their ancestry. Thus both Hungarian and Basque are classified as 'agglutinative languages' although there is no known historical connection between them. (Basque has no known relatives!) Agglutinative languages are languages in which words are in fact not the basic elements that they are with us, but rather are formed by the joining together of smaller elements called 'morphemes'.

Such work has practical consequences. In an agglutinative language, the basic philosophy required to devise a spellcheck will be different from what we would use for English (say), which is not agglutinative. However, if a spellcheck proved successful with Hungarian, then the philosophy underlying it could be adapted for use with Basque. For more on this question, see

http://arxiv.org/abs/cmp-lg/9410004

Jakobson in his turn influenced the anthropologist Claude Lévi- Strauss (1908–), whose friendship with the mathematician André Weil (1906–1998) led to one of the most widely quoted successes of Structural Anthropology. This was a classification of the way in which incest taboos work in traditional societies.

By far the best account of this is the article by my friend and colleague Hans Lausch in *Function*, Vol 4 P3 1980. This is much more accurate and lucid than other more widely available versions, including that in Kemeny, Snell and Thompson's text *Introduction to Finite Mathematics*. I will summarize Lausch's treatment here, although in a somewhat amended form, but readers are urged to try to find the original, which goes much further and provides much more detail than I do.

The structure of a people or tribe obeys six axioms, of which the first five are:

1. Each member of the community belongs to exactly one of n sub-communities, or 'clans', here called $\alpha,\ \beta,\ \gamma,\ \ldots,\ \nu$ for reference;

2. To each clan, α say, is assigned one other (different) clan $W\alpha,$ from which a man of clan α may choose a wife, and women who are not of clan $W\alpha$ may not marry a man from clan α;

3. The children of such a marriage will belong to yet another clan $C\alpha$, different from both α and $W\alpha$;

4. Men from a particular clan may not marry women from the same clan;

5. Children of the same family/tribe whose fathers are from different clans will themselves belong to different clans.

We may then set up an algebra based on the relations W and C. Here W means 'wife of' and C means 'child of'. These two elements have inverses W^{-1}, meaning 'husband of', and C^{-1}, meaning 'father of'. [The stress on the male in these axioms and notations is incidental; we could equally well, with small adjustments, begin with the women.] A string of W's and C's defines a relationship unequivocally. Thus WC^{-1} is 'father's wife' (read the strings from right to left), i.e. 'mother'; CWC^{-1} is 'mother's son or daughter', i.e. 'sibling'; C^3 is 'son's son's child'; and so on.

The sixth and final axiom states that

6. It is the *type of relationship*, not the specific clan, that determines the societal structure. [The relationships between W, C are independent of the values of the letters that follow.]

Thus, if $C^2\alpha = \alpha$ for any one clan α, then $C^2\beta = \beta$ for any other clan, β say. Under these six axioms, the elements W and C generate an algebraic structure known as a 'permutation group'. Different societies conform to different permutation groups, and we can classify the society according to which permutation group it has adopted.

The simplest possible society would have a three-clan structure, and readers might like to see how this would work out. [Up to change of notation, there is only one possibility.] Somewhat more complicated are four-clan societies, of which one example is the Kariera of Australia's Northwest. Here there are the following clans: Karimera, Palyeri, Burung and Banaka. I will term these $\alpha, \beta, \gamma, \delta$ respectively. The clan interrelations are $W\alpha = \beta, W\beta = \gamma, W\gamma = \delta, W\delta = \alpha; C\alpha = \gamma, C\beta = \delta, C\gamma = \alpha, C\delta = \beta$. Knowing these equations enables us to form a complete picture of the kinship system of the Kariera, and readers may care to explore this further. (For example, an α (Karimera) man will have γ (Burung) sons, and their sons in turn will be Karimera, etc.) [One of the defects of the account by Kemeny, Snell and Thompson is that they seem to get Kariera society wrong; they also present their axioms in a way that is at variance with the anthropological literature; and when they come to discuss the Tarau, a tribal society from Manipur state in India, they allow oedipal (mother-son) marriages, which is surely incorrect.]

Notice that here we have no *explanation* of *why* the Kariera adopt this particular permutation group, whereas the Arrerente, for example, adopt another. What we do have, however, is a set of axioms that applies widely to traditional societies and a set of particular cases that describe different societies, but all following the six basic axioms.

The situation is not at all unlike that with which we are familiar in Geometry. We do not really explain *why* the base-angles of an isosceles triangle are equal. Rather we say that it follows directly from the axioms of Euclidean geometry that they are. Or to give a more subtle example, we have geometric axioms that allow different realizations as Euclidean, Lobachevskian or Riemannian geometries, just as different societies end up adopting different permutation groups.

It was this example of Geometry that inspired Thom. When he considered Embryology, as he did on many occasions, he seemed to see the possibility that from the mass of experimental data there could be extracted some basic rules that could form an axiomatic basis for the discipline. The rest would be mathematical deduction.

The goal, he was concerned to stress, is 'the reduction of arbitrariness': we seek, not so much *explanation* as *efficient description*. Such efficient description was envisaged as being mathematical. In fact, Thom is on record as saying that 'the only possible theoretization is mathematical'.

This approach produces a quite different sort of theory from those of Classical Mechanics, for example. In that more familiar case, the underlying assumption is that the quantities under discussion (force, mass, acceleration, etc.) can all be measured and it is the concordance between measured results of experiments and the values predicted by the theory that validates the enterprise. Structuralist theories, by contrast, content themselves with a summary of a given situation (say the clans of Kariera society) and the location of this summary in an overall theoretical system. They are descriptive rather than predictive.

This difference is one that causes controversy. On one occasion, Thom clashed with Francis Crick, the co-discoverer of the structure of the DNA molecule. According to Crick (in his book *What Mad Pursuit*), Thom queried some of Crick's work 'because it did not comport with mathematical theory'. Crick went on to the judgement that

Thom did not understand how Science worked and 'what he did understand he didn't like, and referred to it disparagingly as 'Anglo-Saxon' '.

The incident would seem to have occurred at the fourth Serbelloni conference. Thom's point of view is set out in a footnote to his paper Structuralism and biology. Thom is concerned to develop an analogy between the genetic code and Saussure's linguistic notions. For Thom, 'the present contention of molecular biology, that the genetic code explains morphogenesis [is] ill founded: it amounts to saying that deciphering the alphabet of an unknown language suffices to understand it'.

With the wisdom born of hindsight, we can say that Crick won this particular skirmish. What Thom overlooked was the potential of a code to modify itself. Computer programs provide a clear example. We can embed in the program an instruction to do something or other under certain conditions (IF statements and the like). This corresponds to the power of some genes to turn on and off at certain times in the development of the organism they are encoded for. This is exactly the progress of morphogenesis.

Indeed Thom's example is actually and ironically rather unfortunate. There is a linguistic parallel for exactly the sort of thing he disbelieved, although it comes from diachronic, rather than synchronic, linguistics. But when the alphabet (or more precisely the syllabary) of the ancient language Hittite was deciphered, it became clear that Hittite was an Indo-European language, and this realization enabled it to be translated!

However, the fact that Crick won the battle does not mean that Thom lost the war! The successes that Mathematics achieved with Physics and Astronomy have come about because these areas of study are relatively simple – simple, that is, when we compare them with the biological or the social sciences. The number of variables in these disciplines is so large, and their interactions are so complex, that it is unlikely that simple and accurate predictive models will ever be found to deal with them. If progress is to be made in a mathematical account of such a science, then the general principles of Thom's program will need to be realized in some way or other.