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Problems Section

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year. Solutions to these problems will appear in the next issue of *Parabola*, and if received in time your solution(s) may be used.

Q1201. Let x_1 and x_2 be the solutions of

$$x^2 - (a+d)x + ad - bc = 0.$$

Prove that x_1^3 and x_2^3 are the solutions of

$$x^2 - (a^3 + d^3 + 3abc + 3bcd)x + (ad - bc)^3 = 0. \tag{1}$$

Q1202. Metal bars of length 7.4m each are sold to a builder who wants to cut them into small pieces of two different lengths 0.7m and 0.5m. He needs 1000 pieces of the first length size and 2000 pieces of the second length size, and does not want to spend more money than necessary. Can you advise on the least number of bars he needs to buy, and show him how to perform the cutting?

Q1203. Let a, b and c be the lengths of the sides of a triangle ABC, and x, y and z be the lengths of the three interior angle bisectors. Prove that

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} < \frac{1}{x} + \frac{1}{y} + \frac{1}{z}.$$

Q1204. Assume that x and y are two real numbers satisfying
$4x - 3y = 5$. Prove that

1. $x^2 + y^2 \geq 1$.
2. $4x^2 + 3y^2 \geq \dfrac{25}{7}$.

Q1205. Prove that for any positive integer n there holds

$$1 + \frac{1}{2^3} + \frac{1}{3^3} + \cdots + \frac{1}{n^3} < \frac{5}{4}.$$

Q1206. Let a, b and c be three numbers satisfying

$$a^2 + b^2 + c^2 = 2$$
$$ab + bc + ca = 1.$$

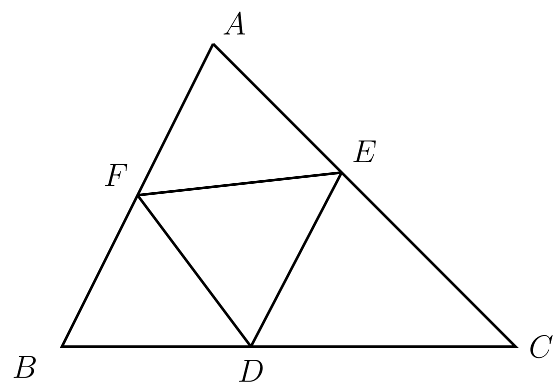
Prove that $|a| \leq \frac{4}{3}$, $|b| \leq \frac{4}{3}$ and $|c| \leq \frac{4}{3}$.

Q1207. Prove that for any real number x there holds

$$x^8 - x^5 + x^2 - x + 1 > 0.$$

Q1208. Given n distinct points on a circle ($n \geq 3$), how many n-polygons having these points as vertices can be constructed?

Q1209. Let D, E and F be three points on three sides of a triangle ABC, as in the figure. Prove that at least one of the triangles AFE, BDF and CED has area not greater than $\frac{1}{4} \times$ area(ABC).



Q1210. If A, B and C are three angles in a triangle ABC, prove that

$$\cos A + \cos B + \cos C \leq \frac{3}{2}.$$

When does the equality hold?