Parabola Volume 42, Issue 3 (2006)

Problems Section

You are invited to submit solutions to any or all of the following problems, accompanied by your name, (school and year if appropriate). Solutions to these problems will appear in the next issue of *Parabola*, and if received in time your solution(s) may be used.

Q1221. (submitted by Frank Drost, Research Associate, School of Mathematics and Statistics, UNSW. Edited.)

Complete the mathematical equations below by inserting the least number of mathematical symbols from the table

$+$	$-$	$\times$	$/$	$\sqrt{}$	!	(	)

on the left hand side of the equation.

$$\begin{array}{ccccc}
0 & 0 & 0 & = & 6 \\
1 & 1 & 1 & = & 6 \\
2 & 2 & 2 & = & 6 \\
3 & 3 & 3 & = & 6 \\
4 & 4 & 4 & = & 6 \\
5 & 5 & 5 & = & 6 \\
6 & 6 & 6 & = & 6 \\
7 & 7 & 7 & = & 6 \\
8 & 8 & 8 & = & 6 \\
9 & 9 & 9 & = & 6 \\
10 & 10 & 10 & = & 6
\end{array}$$

Q1222. How many ways can n different cards be dealt to two persons, given that they may receive unequal numbers of cards but each has at least one card?

Q1223. The three angle bisectors of a triangle ΔABC cut its circumcircle at A_1, B_1 and C_1. Let S be the common area of ΔABC and $\Delta A_1B_1C_1$. Prove that

$$S \geq \frac{2}{3}\text{area}(\Delta ABC).$$

When does equality occur?

Q1224. Let D be a point outside a triangle ΔABC such that A and D are on opposite side of the line BC, and that ΔBCD is equilateral. Prove that for any point M in the same plane with ΔABC

$$MA + MB + MC \geq AD.$$

When does equality occur?
Q1225. (submitted by J. Guest, East Bentleigh, Victoria)
Find all the real roots of

$$3x^5 - 40x^4 + 169x^3 - 271x^2 + 136x - 21 = 0.$$

Q1226. Find the integral value of

$$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\cdots}}}}$$

Q1227. Consider n simultaneous equations in n unknowns $x_1, \ldots, x_n$:

$$\begin{aligned} x_1 + x_2 + x_3 &= 0 \\ x_2 + x_3 + x_4 &= 0 \\ \vdots \quad &= 0 \\ x_{n-1} + x_n + x_1 &= 0 \\ x_n + x_1 + x_2 &= 0 \end{aligned}$$

1. For which values of n do the equations have a unique solution?
2. Find the most general solution when the equations do not have a unique solution.

Q1228. For any real number a, the symbol $[a]$ denotes the integer part of a. E.g.

$$[2] = 2, \quad [3.7] = 3, \quad [-2.4] = -3.$$

Simplify

$$[a] + \left[a + \frac{1}{n}\right] + \left[a + \frac{2}{n}\right] + \cdots + \left[a + \frac{n-1}{n}\right].$$

Q1229. A sequence of polynomials is defined recursively by

$$\begin{aligned} F_1(x) &= \frac{x^2}{2} + \frac{x}{2} \\ F_k(x) &= k\left[\int_0^x F_{k-1}(t)\,dt + x\int_0^{-1} F_{k-1}(t)\,dt\right], \quad k \geq 2. \end{aligned}$$

Find the constant term and the coefficients of x^k and x^{k+1} in $F_k(x)$.
Q1230. Show that the polynomial $F_k(x)$ defined in the previous question satisfies

$$F_k(x) - F_k(x-1) = x^k.$$