Parabola Volume 43, Issue 2 (2007)

Problems Section

You are invited to submit solutions to any or all of the following problems, accompanied by your name, (school and year if appropriate). Solutions to these problems will appear in the next issue of *Parabola*, and if received in time your solution(s) may be used.

Q1241 Show that Simpson's Elementary Rule

$$\int_a^b f(x)dx \approx \left(\frac{b-a}{6}\right)\left(f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right)$$

is an exact equality for the quadratic function

$$f(x) = Ax^2 + Bx + C.$$

Q1242 Use the laws of addition and multiplication of complex numbers (see the first article in this issue) to find the complex numbers z that satisfy

$$z^3 = i.$$

Q1243 Prove that if

$$x^n + y^n = z^n$$

where x, y, z, n are all positive integers and $x \leq y < z$ then it follows that $y > n$.

Q1244 In modular arithmetic if a, b and c are integers then

$$a \equiv b \pmod{c}$$

if $(a - b)$ is an integer multiple of c. Find the integer $n > 0$ that satisfies

$$n(n-1) \equiv n - 2 \pmod{n+1}.$$

Q1245 Estimate the area of a semi-circular disc of radius r using Simpson's Rule

$$\int_a^b f(x)dx = \left(\frac{b-1}{6}\right)\left(f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right).$$

Q1246 A farmer has a dog, a chicken and a bucket of grain to take across a river in a boat. The farmer can only take one item at a time with him in the boat. The dog cannot be left alone with the chicken as it would eat it. The chicken cannot be left alone with the grain as it would eat it.

Find two ways in which the farmer can transport the dog, chicken and grain safely across the river.

Q1247 A game show contestant has a chance to win a prize by guessing which of five boxes the prize is contained in.

After the contestant announces their guess the game show host points to one of the remaining boxes and informs the contestant that the prize is not in this box. The host then asks the contestant if they would like to stick with their original guess or change their selection.

What is the best strategy for the contestant and what is the probability of winning the prize with this strategy?

Q1248 Show that if $f(x)$ is a continuous solution of the functional equation

$$2f(x)f(y) = f(x+y) + f(x-y)$$

then $f(x)$ is an even function, i.e. $f(x) = f(-x)$.

Q1249 One of the most famous numbers is the golden number ϕ which can be written as

$$\phi = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}}.$$

A less famous number is the silver number (or plastic number) p which can be written as

$$p = \sqrt[3]{1+\sqrt[3]{1+\sqrt[3]{1+\cdots}}}\,.$$

Show that

$$\phi = \frac{1+\sqrt{5}}{2}$$

and

$$p = \sqrt[3]{\sqrt{\frac{23}{108}}+\frac{1}{2}} - \sqrt[3]{\sqrt{\frac{23}{108}}-\frac{1}{2}}\,.$$

Q1250 Find the positive integer x that satisfies the identity

$$x = \sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+\cdots}}}}.$$