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# Modelling Global Warming (or Cooling?)

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The Earth's climate is the result of myriad interactions between the Earth's atmosphere and its surface, which is composed of oceans, land masses and ice-caps. A detailed model of these interactions is always at the limits of computing resources, more resources allowing more details to be included. A major difficulty is that the various interactions occur over a great range of different spatial and temporal scales and so detailed models attempting to capture small-scale features have to be enormously large to be able to extend up to large scales. It is perhaps because of this complexity that global warming forecasts of the impact of human generated $CO_2$ emissions within this framework have been met with some scepticism. For example, a petition

http://www.oism.org/pproject/

signed by more than 19,000 American scientists with university degrees in physical science reads

> There is no convincing scientific evidence that human release of carbon dioxide, methane, or other greenhouse gasses is causing or will, in the foreseeable future, cause catastrophic heating of the Earth's atmosphere and disruption of the Earth's climate. Moreover, there is substantial scientific evidence that increases in atmospheric carbon dioxide produce many beneficial effects upon the natural plant and animal environments of the Earth.

Fortunately we do not need to deal with the complexity of coupled atmosphere-ocean-land-ice models to model the basic aspects of global warming. In the following we consider a simple radiation balance model that dates back to an early paper by Arrhenius entitled 'On the influence of carbonic acid in the air upon the temperature of the ground', which was published in *Philosophical Magazine*, Vol 41, (1896). A more modern publication that also takes into account impacts of chemical weathering has been published by Ditlevsen in *International Journal of Astrobiology*, Vol 4, (2005).

The basic model starts with the consideration that solar radiation heats the surface of the Earth which then warms up and radiates heat back out through the atmosphere. The difference between the net incoming solar radiation $R_i$ and the outgoing radiation $R_o$ determines the temperature of the surface of the Earth. The change in mean surface temperature $T$ per unit time $t$ is proportional to this difference, leading to the model equation

$$c\frac{dT}{dt} = R_i - R_o$$

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where $c$ is the heat capacity of the Earth. If $R_i$ and $R_o$ were constants then we could readily solve the above differential equation to obtain

$$T(t) = \frac{R_i - R_o}{c} t + T(0),$$

so that the surface temperature of the Earth would either continually increase $R_i > R_o$, continually decrease $R_i < R_o$ or stay the same, $R_i = R_o$.

However $R_i$ and $R_o$ are not constants but they are functions of the Earth's surface temperature. The temperature dependence of the net incoming solar radiation can be modelled using the equation

$$R_i = \frac{1}{4}(S - \alpha(T)S).$$

In this equation $S$ is the incoming solar radiation and $\alpha(T)$ is the so-called planetary albedo. This is the fraction of incoming solar radiation that is reflected. The factor of one-quarter arises as the ratio of the cross-sectional area of the Earth (which intercepts the solar radiation) to the surface area of the Earth (over which the solar radiation is distributed). The planetary albedo depends on the amount of ice cover on Earth and the amount of cloud cover. Both ice and clouds reflect the solar radiation. The effect of cloud cover is more complicated though because clouds also reflect outgoing radiation. Clearly the lower the temperature the greater the ice cover (until maximum cover is reached) and so a simple mathematical model for $\alpha$ is to assume that $\alpha$ increases linearly with decreasing temperature,

$$\text{i.e.} \qquad \alpha(T) = a - bT$$

where $a$ and $b$ are positive constants. It is useful to express these constants in terms of the parameters: $\alpha_1$, the minimum albedo for an ice-free planet; $\alpha_2$, the maximum albedo for a fully ice-covered planet; $T_1$, the temperature above which the planet has no ice; and $T_2$, the temperature below which the planet is ice-covered. We can then write

$$\alpha(T) = \alpha_1 \Theta(T) + \frac{\alpha_1 - \alpha_2}{T_2 - T_1} \left[ (T_1 - T)\Theta(T - T_1) - (T_2 - T)\Theta(T - T_2) \right]$$

where $\Theta(T)$ is the Heaviside function defined as

$$\Theta(T) = \begin{cases} 0 & \text{if} \quad T < 0, \\ 1 & \text{if} \quad T > 0. \end{cases}$$

The temperature dependence of outgoing radiation can be approximated using the Stefan-Boltzmann Law for blackbody radiation, which gives a temperature dependence proportional to $T^4$. A simple model for the outgoing radiation from the Earth is then given by

$$R_o = \sigma g(T) T^4$$

where $\sigma$ is the Stefan-Boltzmann constant and the factor $g(T)$ is used to represent a temperature-dependent atmospheric greenhouse effect. The greenhouse effect is produced by greenhouse gases in the atmosphere that are transparent to the incoming visible light but absorbing for the outgoing black-body radiation (which has a longer wavelength). The main greenhouse gases are water vapour ($H_2O$) which accounts for about two-thirds of the greenhouse warming and carbon dioxide ($CO_2$) which accounts for most of the other one-third.

Using the temperature-dependent representations of the incoming and outgoing radiation we now have the following model for the change in mean surface temperature with time,

$$\frac{dT}{dt} = \frac{1}{4}(1-\alpha(T))S - \sigma g(T)T^4.$$

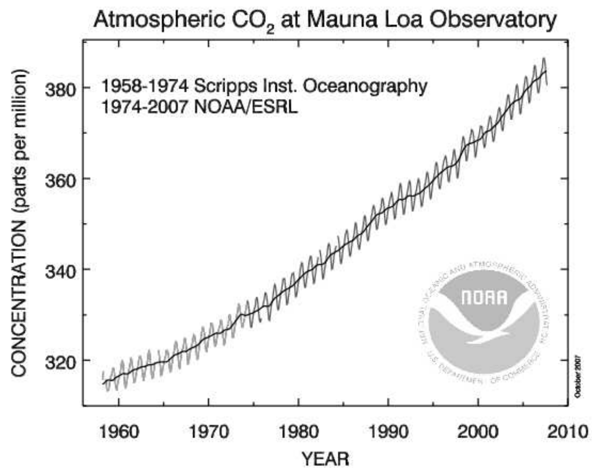
This model has steady state solutions that can be obtained by setting the right hand side to zero. First we consider a simplification of this model that ignores the temperature dependence of the albedo and greenhouse effects,

$$\text{i.e.} \qquad \frac{dT}{dt} = \frac{1}{4}(1-\alpha^\star)S - \sigma g^\star T^4$$

where $\alpha^\star$ and $g^\star$ are constants. This has the steady state solution

$$T = \left(\frac{1-\alpha^\star}{4\sigma g^\star}S\right)^{\frac{1}{4}}.$$

In this model the constant greenhouse term $g^\star$, also known as the emissivity, is defined as the ratio of energy radiated by the Earth to that of the energy radiated by a black body at the same temperature. Estimates of the constants in this simpler model have been given as $S = 1367 Wm^{-2}$, $\alpha = 0.3$, $\sigma = 5.767 \times 10^{-8} JK^{-4}m^{-2}s^{-1}$ and $g^\star = 0.612$, yielding a mean surface temperature of $T = 287K = 15C$. If there was no greenhouse effect then we would have $g^\star = 1$ and $T = 254K = -19C$. The difference between these two temperatures is called the greenhouse temperature difference. The current greenhouse temperature difference is $\Delta T_g = 34K$. As this is a temperature difference we could equivalently write $\Delta T_g = 34C$, i.e. the greenhouse effect makes our planet $34C$ warmer then it would otherwise be – atmospheric water vapour and atmospheric carbon dioxide are necessary to sustain life on Earth. The simple mathematical model can be used to explore how sensitive the mean surface temperature is to the greenhouse effect. For example if the current value of the emissivity was halved to $g^\star \approx 0.3$ this would lead to an uninhabitable planet with a mean surface temperature of $T = 343K = 70C$. To go beyond this analysis we need an explicit model for the temperature dependence of $g$ but this is complicated by many feedback processes. One of the major feedback effects involves $H_2O$. If the climate cools then atmospheric water vapour decreases resulting in a smaller greenhouse temperature difference (more cooling) and thus less water vapour and so on. If the climate warms then there is more water vapour causing more warming and so on. The feedback effects involving $CO_2$ are based on two carbon cycles: an organic carbon cycle involving photosynthesis, which leads to less $CO_2$, and respiration and decay, which leads to more $CO_2$. There is also an inorganic carbon cycle involving silicate weathering, which leads to less $CO_2$, and carbonate metamorphism, which leads to more $CO_2$. These cycles, which occur over very different timescales, have kept the planet habitable. A major concern at the moment is the increase in $CO_2$ due to human industrial activities. This has lead to a large buildup of $CO_2$ in the atmosphere because there is no restoring balance to compensate this increase. Current efforts to stimulate large-scale planting of trees is all about trying to create a restoring balance. The figure below shows measurements of $CO_2$ levels in the atmosphere downloaded from www.esrl.noaa.gov/gmd/. The

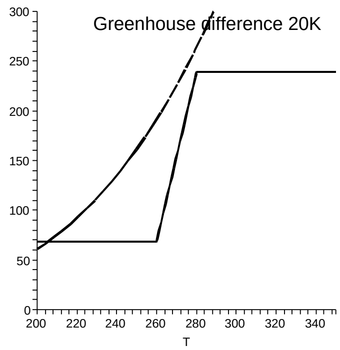


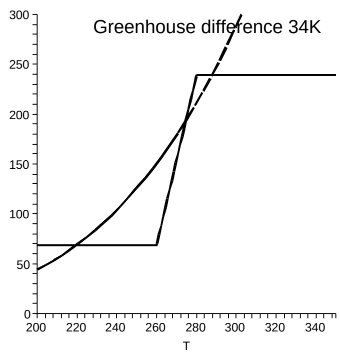

increase in $CO_2$ levels has been linked to human industrial activity and it is generally agreed that levels have risen about 100 parts per million since pre-industrial times. Thus about one-quarter of the greenhouse $CO_2$ gases are due to human activity and given that $CO_2$ gases represent about one-third of total greenhouse gases, we have that about one-twelfth of the greenhouse effect is due to human activity. A simple minded calculation is to assume that the emmissivity has been reduced by one-twelth due to human activity, i.e. in the absence of human activity $g^\star = \frac{12}{11}0.612$, with the corresponding steady state temperature of $T^\star = 281K$. This simple minded calculation suggests that the Earth's surface is about $6K$ or $6C$ warmer due to industrial activity. This is in conflict with the measured warming of $0.8K$ over the past one hundred years but the implicit assumption that emmissivity scales linearly with the concentration of $CO_2$ is incorrect. Just how much the mean surface temperature would rise with rising carbon dioxide levels has been the subject of much discussion, analysis and debate in the scientific literature. The temperature change associated with a doubling of $CO_2$ has been defined as a climate sensitivity parameter and reports from the Intergovernmental Panel on Climate Control (IPCC) usually quote a range from $1.5K$ to $4.5K$ for this sensitivity.

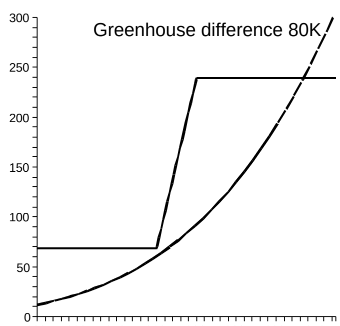
One way for us to gain further insight into the greenhouse effect is to write

$$g(T) = \left(\frac{T - \Delta T_g}{T}\right)^4$$

so that the greenhouse effect temperature difference $\Delta T_g$ now appears as a parameter in the model. The different possible steady state solutions for the mean surface temperature can now be found by plotting curves of $R_i(T) = \frac{1}{4}(1 - \alpha(T))S$ versus $T$ and $R_o(T) = \sigma g(T)T^4$ versus $T$ for different values of the parameter $\Delta T_g$. The temperatures at which these curves intersect define different possible steady states. The figure below shows a plot of $R_i(T)$ versus $T$ and $R_o(T)$ versus $T$ for three parameter values $\Delta T_g = 20K, \Delta T_g = 34K, \Delta T_g = 80K$. Recall that the current situation is modelled with $\Delta T_g = 34K$.




In the plot with $\Delta_g = 20K$ there is a single intersection point at $T_I = 206K = -67C$. This is a stable steady state solution because if $T > T_I$ then $R_o(T) > R_i(T)$ so that $\frac{dT}{dt} < 0$ and $T$ will decrease back towards $T = T_I$. Conversely if $T < T_I$ then $R_o(T) < R_i(T)$ so that $\frac{dT}{dt} > 0$ and $T$ will increase back towards $T = T_I$. This steady state solution would correspond to a frozen snowball Earth.

In the plot with $\Delta_g = 80K$ there is again a single intersection point, this time at $T_{II} = 333K = 60C$. This too is a stable steady state solution and it represents an uninhabitable planetary solution. It whould not be as uninhabitable as Venus where the dense atmosphere has resulted in a large greenhouse effect with a mean surface temperture of about $735K = 462C$.

The plot with $\Delta_g = 34K$, representing present-day conditions, is very interesting with three steady state solutions $T_a = 219 = -54C, T_b = 274K = 1C, T_c = 287K = 15C$. It is easy to see, using arguments similar to those above, that both $T_a$ and $T_c$ are stable solutions whereas $T_b$ is an unstable solution. We are currently enjoying the stable solution at $T_c = 287K$, but this stability is based on many simplifying assumptions.

The model for mean Earth surface temperatures described above is a very simple mathematical model that can be used to introduce senior secondary school students to global warming. This simple energy balance model captures the fundamental relationship between the greenhouse effect and the mean Earth surface temperature but it has nothing to say about how surface temperature changes impact on the Earth's climate, except in the very extreme cases of a snowball Earth or a hothouse Earth. Current research on the relationship between mean surface temperatures and climate is based on the simulation and analysis of very complicated coupled ocean-atmosphere models. These models are starting to reveal that even small mean surface temperature changes of just one degree could have dangerous climate consequences leading to regional changes that would be far outside the range of experience of ecosystems, wildlife and humans in that local region. The detailed study of this by Hansen and co-workers, published in *Atmospheric Chemistry and Physics*, Vol 7 (2007), can be downloaded from the NASA website `http://www.giss.nasa.gov`. The CSIRO website `http://www.csiro.gov.au` is also worth visiting for further information on climate modelling. The findings of Hansen and co-workers stand in stark contrast to the words quoted from the petition referred to at the start of this article. It is fitting to conclude by balancing those words with a quote from Hansen's paper:

> In the nuclear standoff between the Soviet Union and United States, a crisis could be precipitated only by action of one of the parties. In contrast, the present threat to the planet and civilization, with the United States and China now the principal players, requires only inaction in the face of clear scientific evidence of the danger.