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Problems Section

You are invited to submit solutions to any or all of the following problems, accompanied by your name, (school and year if appropriate). Solutions to these problems will appear in the next issue of *Parabola*, and if received in time your solution(s) may be used.

Q1271 (suggested by Julius Guest, Victoria)

Solve simultaneously

$$x^2 + xy + y^2 = 189$$
$$x - \sqrt{xy} + y = 9.$$

Q1272 Find all whole numbers such that when the third digit is deleted, the resulting number divides the original one. (For example, 34 divides 340.)

Q1273 Find all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying $f(0) = 1$ and

$$f(x - y) = \frac{1}{2}f(a - y)f(x) + \frac{1}{2}f(a - x)f(y)$$

for all $x, y \in \mathbb{R}$ and a given $a \in \mathbb{R}$.

Q1274 Does there exist a natural number n such that the fractional part of the number $(2 + \sqrt{2})^n$ exceeds 0.999999?

Q1275 Show that for all integers $n > 2$,

$$2^{n(n-1)/2} > n!$$

Q1276 If b_1, b_2, c_1 and c_2 are real numbers such that

$$b_1 b_2 = 2(c_1 + c_2),$$

show that at least one of the equations

$$x^2 + b_1 x + c_1 = 0$$
$$x^2 + b_2 x + c_2 = 0$$

has two real roots.

Q1277 Four squares are constructed from 4 sides of a convex quadrilateral $ABCD$ such that the squares are all in the exterior of $ABCD$. Let S be the area of $ABCD$. Show that the area S' of the quadrilateral formed by the centres of the squares satisfies $S' \geq 2S$. When does equality occur?

Q1278 Find all prime numbers p such that $(2^{p-1}-1)/p$ is a perfect square.

Q1279 One hour after leaving home, a car broke down and had to travel the remaining trip at $3/5$ of its original speed. The car arrived 2 hours late. If the incident had occurred 50 km further along, the car would have arrived 40 minutes sooner. What is the original speed of the car?

Q1280 Show that in a triangle the median corresponding to the longest side is shorter than that side.