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Problems 1281–1290

Q1281 Prove that for any real numbers a and b there holds

$$\frac{1+|a|}{1+|b|} \le 1+|a-b|.$$

Q1282 Let a and b be two real numbers satisfying $0 \le a, b \le 1/2$ and $a+b>0$. Show that

$$\frac{ab}{(a+b)^2} \le \frac{(1-a)(1-b)}{(2-a-b)^2}.$$

Q1283 Generalise the result of **Q1282** to the case of three numbers a, b and c.

Q1284 Let A_1, B_1, and C_1 be three points on the sides BC, CA and AB (respectively) of a triangle ABC. Show that

$$\frac{AC_1}{C_1B}\,\frac{BA_1}{A_1C}\,\frac{CB_1}{B_1A} = \frac{\sin\angle ACC_1}{\sin\angle C_1CB}\,\frac{\sin\angle BAA_1}{\sin\angle A_1AC}\,\frac{\sin\angle CBB_1}{\sin\angle B_1BA}.$$

Q1285 Let ABC be a triangle such that sides AB and AC are fixed, but the angle $\angle BAC$ may vary. From the exterior of ABC, construct 3 squares $ABDE$, $ACGF$ and $BCHK$. Find the angle $\angle BAC$ such that the area of the hexagon $DEFGHK$ is maximum.

Q1286 From the exterior of a triangle ABC, draw 3 equilateral triangles ABX, BCY and CAZ, whose corresponding centroids are M, N and K. Show that MNK is an equilateral triangle.

Q1287 Let A and B be two points on the parabola $y = x^2$. Assume that $AB = 2$. Find the positions of A and B such that the area of the region formed by AB and the parabola is maximum.

Q1288 Show that there are no integers x and y satisfying

$$x^2 - 2y^2 = 5.$$

Q1289 Two chess teams play against each other in a competition. The teams have different numbers of players, and one team has an odd number of players. Each player has to play one game with each player of the other team. The total games played are 4 times the total players of both teams. How many players has each team??

Q1290 Prove that from any 4 real numbers we can choose x and y satisfying

$$0 \leq \frac{x-y}{1+xy} \leq 1.$$