PROBLEM SECTION

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions and the names of those who submit solutions by March 31, 1975, will be published in the next issue Vol. 11 No. 2!

Although the problems are unclassified, the first 4 problems should require no more mathematical knowledge than is learnt in first or second form and the next 4 problems should require no more mathematical knowledge than is learnt in the first 4 forms of High School. Senior students are thus encouraged to attempt the later problems.

261. In a right-angled triangle, the shortest side is a cm long, the longest side is c cm long and the other side is b cm. If a, b, c are all integers, when does $a^2 = b + c$? (This is question 3 of Mr White's article "Liethagoras theorem" in this issue.)

262. On his birthday in 1975 John reaches an age equal to the sum of the digits in the year he was born. What year was that?

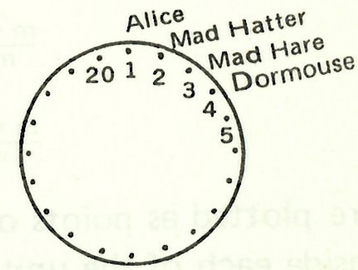
263. Three hundred soldiers are positioned in 15 rows each containing 20 soldiers. From each of the 20 columns thus formed the shortest soldier falls out and the tallest of these 20 men proves to be private Jones. They then resume their places on the parade ground. Next the tallest soldier in each row falls out, and the shortest of these 15 soldiers is private Smith. Who is taller, Jones or Smith?

264. In the 1974 cricket XI there were 7 boys who had been in the 1973 XI, and in the 1973 XI there were 8 boys who had been in the 1972 XI. What is the least number who have been in all three XI's?

Answer the same question with x instead of 7 and y instead of 8. For what values of x and y is it possible that there were no boys in all three XI's?

265. If you are required to make an exact copy of an irregular hexagon given a ruler and a protractor, what is the least number of measurements you would have to make? If you had no protractor could you still do it? If so would a greater number of measurements be needed?

What would be the least number of measurements required to copy an irregular polygon with n sides?



266. At the mad hatter's afternoon tea party there are twenty seats, 4 neighbouring ones with red cushions (1, 2, 3 and 4 in the diagram) being initially occupied by Alice, the mad hatter, the march hare and the dormouse. Instead of all moving round one seat at a time (as in the classical story) the members of the party move quite independently as the fancy takes them, but always to an unoccupied seat 7 places away in either direction. Even the dormouse proves to be wakeful enough to carry out this complicated manoeuvre several times. At a later time it turns out that they are again sitting next to one another on the same red-upholstered chairs (1, 2, 3 and 4), though none is in the same place as initially. How many possible seating arrangements are there at the finish and what are they?

267. A chain has 2047 links in it. It is to be separated into a number of pieces by cutting and disengaging appropriate links, in such a way that any number of links (from 1 to 2047) may be gathered together from the parts of chain thus produced. What is the smallest number of links which must be cut to achieve this?

(For example, if the chain had 7 links it would have sufficed to disengage 1 link, the third from an end, producing pieces of chain with 1 link, 2 links and 4 links. You can easily check that any number of links up to 7 can be gathered using these pieces.)

268. Prove that $11^{10}-1$ is divisible by 100.

269. (i) Show that for any positive integer n

$$2 < (1+\tfrac{1}{n})^n < 3.$$

(ii) Which is larger, 1000^{1000} or 1001^{999}?

270. Find all positive integers between 1 and 100 having the property that $(n-1)!$ is not divisible by n^2.

271. Prove that if the sum of the fractions $\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2}$ (where n is a positive integer) is put in decimal form, it forms a non-terminating decimal which is periodic after several terms.
(e.g. For $n = 3, \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{47}{60} = .78\dot{3}$ is periodic after 2 decimal places.)

272. Let m and n be two relatively prime positive integers. Prove that if the $m + n-2$ fractions

$$\frac{m+n}{m}, \frac{2(m+n)}{m}, \frac{3(m+n)}{m}, \ldots, \frac{(m-1)(m+n)}{m},$$

$$\frac{m+n}{n}, \frac{2(m+n)}{n}, \frac{3(m+n)}{n}, \ldots, \frac{(n-1)(m+n)}{n},$$

are plotted as points on the real number line, exactly one of these fractions lies inside each of the unit intervals (1,2), (2,3), (3,4), . . . , $(m+n-2, m+n-1)$. (e.g. If $m = 3$, $n = 4$, then 7/4 is between 1 and 2, 7/3 is between 2 and 3, 14/4 is between 3 and 4, 14/3 is between 4 and 5, and 21/4 is between 5 and 6.)

Solutions to Problems J251–O260 (Vol. 10 No. 3)

Junior

J251 Farmer Jones grew a square number of cabbages last year. This year he grew 41 more cabbages than last year and still grew a square number of cabbages. How many did he grow this year?

Answer: If Farmer Jones grew x^2 cabbages this year, and y^2 last year then $x^2-y^2 = 41$.

i.e. $(x-y)(x+y) = 41$. Since the only factorisation of 41 into two integer factors is 1×41 we must have $x-y = 1$, $x + y = 41$, and solving gives $x = 21$, $y = 20$. Hence $x^2 = 441$.

J252 I met triplets, A, B and C whose names were John, Peter, and Mick. When I asked who was who, A answered, "I'm not Peter."

B said, "I'm Peter."

C said, "I'm not John."

Then they told me that only one of them was telling the truth. Who was who?