YOUR LETTERS

Dear Sir,

I have a problem regarding periodic decimals. It concerns a decimal such as $0.4\dot{9}$ (i.e. 0.4999 . . .). If we let $x = 0.4\dot{9}$, then $10x = 4.9\dot{9}$ and so

$$9x = 10x - x = 4.9\dot{9} - 0.4\dot{9} = 4.5$$

Thus $x = ½$, or 0.5 in decimal form.

In general, the decimal $0.a_1 a_2 \ldots a_n \dot{9}$ may also be written $0.a_1 a_2 \ldots (a_n + 1)$. Since this second decimal may be taken as any terminating decimal (between 0 and 1), it follows that every decimal can be expressed as a periodic one.

This problem follows question 271 in Parabola Vol. 11 No. 1. The above method implies that any fraction of the form p/q where $p < q$ can be represented as a periodic decimal after a certain number of terms. Of course, there is no difference if $p > q$.

Greg Middleton

Marist Brothers, North Sydney

Solution

Greg is right in stating that

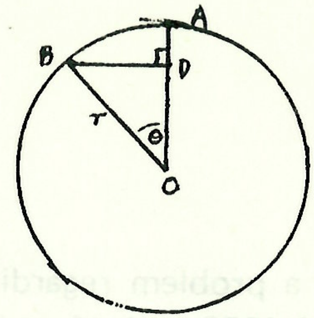
$$0.a_1 a_2 a_3 \ldots a_n \dot{9} = 0.a_1 a_2 \ldots a_{n-1} (a_n + 1).$$

This is usually stated by saying that, although decimals define real numbers, they do not define them uniquely. The problem is overcome by excluding $\dot{9}$ from the end of a decimal and then saying that every decimal (i.e. every decimal which does not end in $\dot{9}$) defines a unique real number. Thus problem 271 is still a problem!

Dear Sir,

A friend of mine reckons the earth, being a circle, drops 6 or 7 inches each mile. I tried to work this out exactly but cannot get a final answer to suit our argument. I'd very much appreciate if you could give me the answer to this problem.

Jimmy Pike



Solution

If O is the centre of the earth, then

$$\text{drop (in miles)} = AD = AO - OD = r - r\cos\theta$$

where r miles is the radius of the earth and θ is the angle at O between A and B: thus $r \cong 3961$.

Now $$1 \text{ mile} = \text{arc } AB = r\theta$$

So $$\theta = \frac{1}{3961} \cong 0.00025 \text{ radians.}$$

If you look up your cosine tables, you will find that this value of θ is too small to give any value for $\cos\theta$.

However, there is another result which says that for small values of θ, $\cos\theta$ is approximately the same as $1-\frac{1}{2}\theta^2$.

Thus $$\text{drop (in miles)} = r(1-\cos\theta) \cong \tfrac{1}{2}r\theta^2 = \frac{1}{2r}$$

So $$\text{drop (in inches)} \cong \frac{63{,}360}{2 \times 3{,}961} \cong 8 \text{ inches.}$$

[If other readers have similar problems, I will be prepared to answer them – Ed.]

Dear Sir,

A very similar problem to Problem 0256 (Vol. 10 No. 3) that I saw a few years ago is to evaluate the infinite product:–

$$(1 + 10^{-1} + 10^{-2} + 10^{-3} + \ldots + 10^{-9})(1 + 10^{-10} + 10^{-20} + \ldots + 10^{-90})$$
$$(1 + 10^{-100} + \ldots + 10^{-900}) \ldots .$$

This of course simplifies to $1\frac{1}{9}$.

We can now see a way of representing the sum of some geometric series as a product. I imagine this could be very useful, but I have never seen it done anywhere. Just in passing, a more complete generalization of this sort of problem is shown below:

Simplify $$(1 + n^{-1} + n^{-2} + \ldots + n^{-m})(1 + n^{-(m+1)} + n^{-2(m+1)} + \ldots +$$
$$+ n^{-m(m+1)})(1 + n^{-(m+1)^2} + \ldots + n^{-m(m+1)^2}) \ldots .$$

This expression can either be given a final term, or can be continued to infinity, as is shown here. When it is continued to infinity, the value of the expression is

$$1 + \frac{1}{n} + \frac{1}{n^2} + \ldots + \frac{1}{n^m} + \ldots = \frac{n}{n-1}.$$

If, in the general problem, n is given the value 3, m is given the value 1, we obtain the problem given in Parabola, and n = 10, m = 9 gives the similar problem I gave above.

John Reeves
Scots School, Bathurst (1974)

☆ ☆ ☆

Doodling

A bright young fellow was fiddling with pencil and paper. He wrote:

$$5 \times 4 \times 3 \times 2 \times 1.$$

Looking at this he thought, "This number is divisible by 5". He then wrote down:

$$4 \times 3 \times 2 \times 1$$

and noticed that if he added 1 to this, the number would still be divisible by 5. Perhaps he had stumbled upon something! He tried it for 7. He wrote:

$$6 \times 5 \times 4 \times 3 \times 2 \times 1 + 1$$

and discovered it was divisible by 7. Things were warming up! Imagine his disappointment when he tried it for 4.

$$3 \times 2 \times 1 + 1$$

is not divisible by 4.

By this time the poor fellow was rather tired. He speculated that his theory worked for odd numbers, only, and left it at that. Was he correct?

W.J. Ryan,
Mitchell College of Advanced Education