

MATHEMATICAL GAMES

Mastermind

Mastermind is a game which goes by several different names, and you may already know it, perhaps as "bulls and hits". It is a beautiful game to while away those end-of-year free periods (if your teacher is co-operative).

Mastermind is a game essentially for two people, although each "person" could be a small team (especially if the other player is your maths teacher). One player writes down a number out of sight of the second player, who then attempts to guess the number. The object of the game is to take as few guesses as possible to find the number. The players then change places and the process is repeated, the one who took the fewer guesses being the winner. There is some luck involved, but some reasoning skill is needed.

In the simplest version of the game, the number written down, and the guesses made, are restricted in the following way: the numbers consist of four different digits, and the first digit may not be 0. After each guess the "mastermind" tells the "code-breaker" the number of "bulls" (correct digits in the correct position) and the number of "hits" (correct digits but in the wrong position) that he has obtained. Thus, if the number were 4537, and the guess 1234, the response would be "1 bull" (for the 3) "1 hit" (for the 4). Note that the 3 is not counted as a hit.

In other versions of the game, the number of digits may be more than four, and digits may be repeated. However, this causes some problems of definition. If the number is 1123 and the guess is 4511, is the correct response "2 hits" or "4 hits"? If the number is 1123 and the guess is 1411, is the correct response "1 bull, 1 hit", "1 bull, 2 hits" or what?

A full game of the simple version may proceed as follows:

| | |
|------|-----------|
| 1234 | "4 hits" |
| 2341 | "4 hits" |
| 4123 | "4 hits" |
| 3412 | "4 bulls" |

However, this would be an unusually quick game. You should be able consistently to guess the number in six or eight goes.

Some Strategy: Your first guess is quite random — or it would be if 0 had the same probability of appearing as the other digits (see questions at end). So you are faced with the problem whether to include 0 in your first guess, and you are told "4 hits" (lucky!). It is not at all clear what it is best to do. In the above game, the four digits were simply cycled to the left. When the response was "4 hits" again, there were only two numbers consistent with both responses, namely

4123 and 3412, so the game soon finished. But if the response to 2341 had been "1 bull, 3 hits" or "2 bulls, 2 hits", it is not clear (to the writer) what is best to do. A different strategy after a "4 hits" response is to introduce two new digits. Thus, after 1234 "4 hits", you try 2156 to which the response must be "2 bulls", "2 hits" or "1 bull, 1 hit". The first of these responses suggests the third guess 2143 "4 bulls". The second could require a total of 5 guesses:

| | |
|------|-------------------|
| 1234 | "4 hits" |
| 2156 | "2 hits" |
| 3412 | "2 bulls, 2 hits" |
| 3421 | "4 hits" |
| 4312 | "4 bulls" |

The third could require (at worst) 6 guesses:

| | | |
|------|------|------------------|
| e.g. | 1234 | "4 hits" |
| | 2156 | "1 bull, 1 hit" |
| | 2413 | "1 bull, 3 hits" |
| | 2341 | "4 hits" |
| | 3142 | "1 bull, 3 hits" |
| | 4123 | "4 bulls" |

Is the best strategy after "4 hits" to do as I have done, introduce 2 new digits?

You may like to think about the best strategy but whether you do or not, I'm sure you'll have a lot of fun playing Mastermind.

Finally, there are attractive sets of Mastermind (in several versions) available in some shops, which you may like to look at.

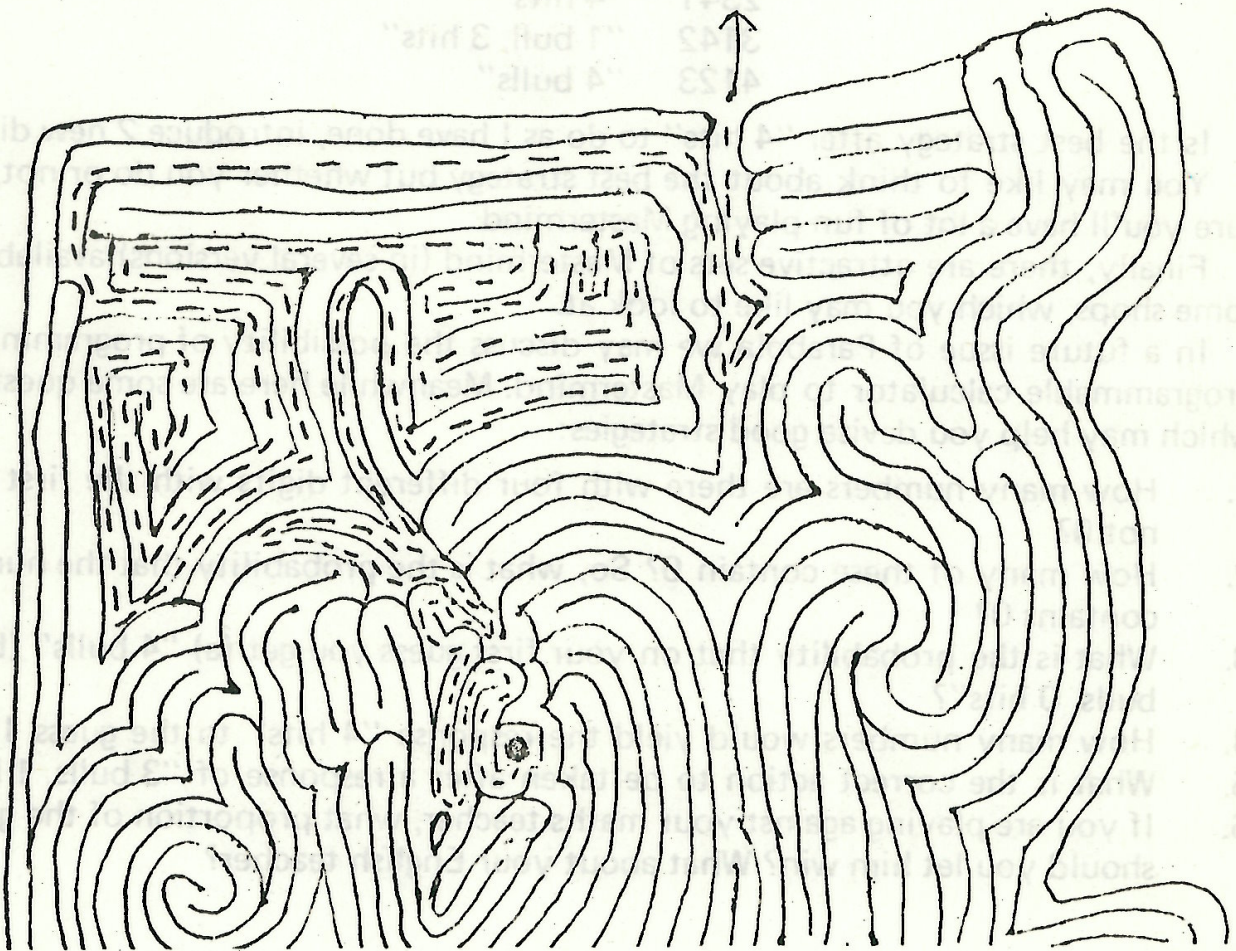
In a future issue of Parabola we may discuss the possibility of programming a programmable calculator to play Mastermind. Meanwhile here are some questions which may help you devise good strategies:

1. How many numbers are there with four different digits with the first digit not 0?
2. How many of these contain 0? So, what is the probability that the number contains 0?
3. What is the probability that on your first guess you get (a) "4 bulls" (b) "0 bulls, 0 hits"?
4. How many numbers would yield the response "4 hits" to the guess 1234?
5. What is the correct action to be taken after a response of "3 bulls, 1 hit"?
6. If you are playing against your maths teacher, what proportion of the games should you let him win? What about your English teacher?

Mazes

The solution to the maze in the last issue of Parabola is given below. You were also asked to see if you could devise a theory to help you find your way out of mazes similar to the one given. One method (although not the shortest) of getting out of that maze is to keep to the left hand side of all the passages, turning left every time you come to a branch in the passage. If this does not work on a maze, an obvious alternative would be to keep to the right hand side of all passages. If neither of these works, a modification is to use the "left hand" method until you find yourself at any point in the maze twice (the first such point will be a branch) and then switch to the "right hand" method, switching back again if you return to some point again, and so on.

In this issue, a second maze has been provided for you, but you may find the above theory will not work!



**Solution to maze in Vol. 11 No. 2
(Only part of the maze is shown for clarity)**