Parabola Volume 11, Issue 3 (1975)

SCHOOL MATHEMATICS COMPETITION 1975 – EXAMINER'S REPORT

Junior Division

Question 1: In fact, the two numbers must each end in zero (see solution in last issue of Parabola), but all who got the key fact that, if a digit x appeared in one number, then 9–x must also appear in the number (except where x is the last non-zero digit) were heavily rewarded.

Question 2: A large number of candidates got part (a) right, as far as labelling the triplets possible or impossible was concerned, but gave no reasons for their answers. It is always pointed out on the front of the examination paper that "the methods you use to get answers are as important as the answers themselves". Far fewer got part (b) correct or half-correct (there were 2 conditions necessary to *guarantee* that (a,b,c) would be impossible). However, many of the "half-correct" students were only wrong on the other condition because they understandably got muddled over whether to use a or a + 1 and so on.

Question 3: Part (a) was done successfully and well by a considerable number of candidates. I felt that I could see the competitors thinking their way to the proof in what they wrote. There was also a pleasing variety of proofs used. I think that only the 1st Prizewinner got part (b) right, though.

It was a bit disturbing to find that many candidates thought that "positive" meant "even".

Question 4: In my opinion this was the hardest question on the paper, to get 100% right. To prove your construction right, you did have to prove that the three circles concerned were concurrent – unless you spotted that P must be the orthocentre and worked backwards. Many students assumed, quite unjustifiably, that P was the circumcentre of ABC.

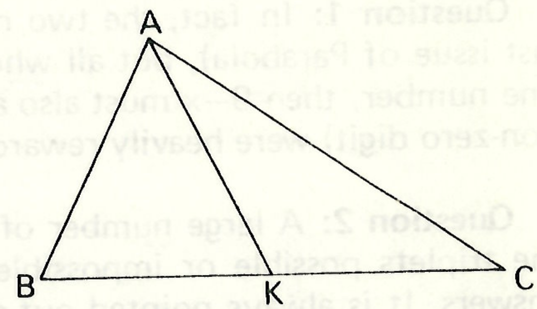
Question 5: I thought this question was a test of one's ability to see the cube in one's mind and then to be able to turn it round, after first nailing it to the floor.

We admit that the question could have been more clearly worded, but most competitors seemed to understand what we meant, even though their answers were wrong for the most part. 720 or 120 were the most frequent (wrong) answers.

Senior Division

Question 1 was found to be easy by many candidates, and question 2, whilst harder was also straightforward.

Question 3: Various candidates made reasonable attempts to part (a) by continuity arguments as follow:



Consider an isosceles triangle ABK where K is a point on BC with AB* = AK*. The point C is now allowed to vary on the line BK. The case when C is very distant from K is examined first, and it is seen that the bisector and median cannot cross over as C varies on the far side. It was then argued that the median was the longer of the two. The perpendicular is obviously always the shortest.

Question 4: Most candidates found this too abstract.

Question 5: It should be noted that (c) and (d) can (and should) be deduced from the formulae (a) and (b) only, whether or not you can prove them. This is now a problem of mathematical induction (which (a) and (b) certainly are not).

Note that there are two types of mathematical induction:

Let $P(n)$ be a property of the positive integer n, with $P(1)$ true.

If we can show that either

(1) $P(n-1)$ true implies $P(n)$ true; or

(2) $P(r)$ true for all $r < n$ implies $P(n)$ true

we may assert that $P(n)$ is true for all n.

Here we are using process (2), not process (1).

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