PROBLEM SECTION

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions, and the names of those who submit solutions before the publication of the next issue, will be published in Vol. 12 No. 1.

Although the problems are unclassified, the first 4 problems should require no more mathematical knowledge than is learnt in first or second form and the next 4 problems should require no more mathematical knowledge than is learnt in the first 4 forms of High School. Senior students are thus encouraged to attempt the later problems.

The first five questions are taken from the Wisconsin Talent Search conducted by the University of Wisconsin, U.S.A., to find budding young mathematicians in high schools in America. This is similar to the competition conducted in July each year at the University of N.S.W.

285. C.F. Gauss was given the problem of summing the numbers from 1 to 100 when he was a student. He did it this way:—

$$S = 1 + 2 + 3 + \ldots + 100, \text{ then}$$
$$S = 100 + 99 + 98 + \ldots + 1, \text{ so}$$
$$2S = 101 + 101 + 101 + \ldots + 101 = 100 \times 101, \text{ or } S = 5{,}050.$$

We pose the harder problem of summing all the numbers from 1 to 300 which are multiples of 3 or 5 or 7. That is, calculate

$$S = 3 + 5 + 6 + 7 + 9 + 10 + 12 + 14 + 15 + 18 + \ldots + 300.$$

286. Show how to cut up and re-assemble five squares of side length 1 into a single square.

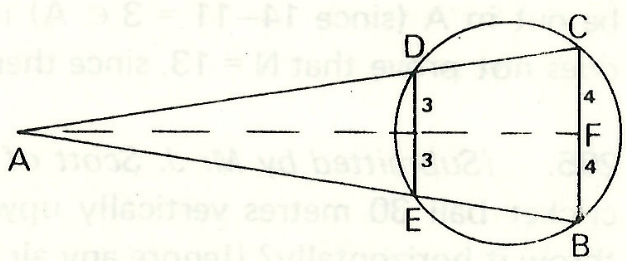
287. Show that $\sqrt{(1976^{1977} + 1978^{1979})}$ is irrational.

288. Find *all* solutions of the equations with 13 unknowns:

$$x_1x_2 = x_2x_3 = x_3x_4 = \ldots = x_{12}x_{13} = x_{13}x_1 = 1.$$

Solve similar equations for 12 unknowns.

289. In a circle of radius 5 cm we have two parallel chords CB and ED of lengths 8 cm and 6 cm respectively. CD and EB are extended to meet at A. Calculate the altitude of the triangle ABC: that is, the legnth of AF.



290. If x_1, x_2, x_3, x_4, and x_5 are all positive numbers, prove that

$$(x_1 + x_2 + x_3 + x_4 + x_5)^2 \geqslant 4(x_1x_2 + x_2x_3 + x_3x_4 + x_4x_5 + x_5x_1).$$

291. Among 11 apparently identical metal spheres, 2 are radioactive. We have an instrument which detects the presence of radioactivity. Show that it is possible to determine the radioactive spheres after 7 uses of the instrument.

292. The plane is divided (like a chessboard) into congruent squares. A finite number of squares are coloured black, the others (infinitely many) remain white. After 1 second, the squares change their colour according to the following rule: if the upper and right neighbours of a given square, S, have the same colour then S takes this colour (irrespective whether it had this colour already or not); if they have opposite colours then the colour of S remains unchanged. This process is repeated after 2 seconds, 3 seconds, ... Describe the eventual colouring of all squares, and prove your assertion.

293. Determine all positive integers n such that 1! + 2! + 3! + . . . + n! is a perfect square.

294. Given a positive integer n, find (in terms of n) the largest integer N such that the set of integers S = {n, n+1, n+2, . . . N} can be split up into two subsets A and B such that $A \cup B = S$ and the difference x–y between any two elements x,y of one of the sets A,B is in the other set.

E.g. if n = 3, $N \geqslant 13$ since the set S = {3, 4, 5, . . . 13} can be split into the subsets A = {3, 4, 5, 11, 12, 13} and B = {6, 7, 8, 9, 10}. Note that 14 cannot be put in A (since $14-11 = 3 \in A$) nor in B (since $14-7 = 7 \in B$). However, this does not prove that N = 13, since there may be another way of choosing A and B.

295. *(Submitted by Mr J. Scott of Barker College)* A man is able to throw a cricket ball 30 metres vertically upwards. What is the furthest distance he can throw it horizontally? (Ignore any air resistance).

296. A parallelepiped is a solid figure with six faces each of which is a parallelogram. You are given four points, A, B, C, D in space not all lying in the same plane. How many parallelepipeds exist with A, B, C, D included amongst the eight vertices?

Solutions to Problems 273 – 284 (Vol. 11 No. 2)

273. What is the smallest and largest possible number of Fridays that can occur on the 13th of a month in any calendar year (e.g. Friday 13th June is the only one in 1975).

Answer: The 13th January can be a Sunday (Su), Monday (M), Tuesday (Tu), Wednesday (W), Thursday (Th), Friday (F), or a Saturday (Sa). The corresponding days of the week for the remaining months is set out in the following table: