MATHEMATICAL GAMES

Computer Art Without a Computer

Last year we presented you with mazes which are not strictly a game, and similarly in this issue we are presenting you with a method for constructing mathematical patterns. Art and mathematics have been closely related all through the ages, so it is not surprising to learn that computers are used to generate these patterns. One form of this "computer art" consists of dots or lines arranged in a fashion which is partly random and partly controlled: the computer generates random numbers and then applies a set of rules to these numbers to determine the final pattern. However, all of this can be done without a computer, and in this article I describe one method of constructing such patterns. All that is needed is a supply of random digits (0-9); these can be obtained by opening log tables and with your eyes shut, stabbing with your finger and taking the last digit of the number you hit. Alternatively, use the telephone directory, taking the final digits of telephone numbers.

Starting the Pattern

Make a copy of the grid (over page) in dark ink, and clip a thin piece of paper over your copy. You should be able to see the grid through the paper.

In explaining the pattern, we will talk as though we are playing a game. A move consists of drawing an interval along the grid from one point to another, each move being made at a 90° turn to the right from the previous move as shown in Figure 1.

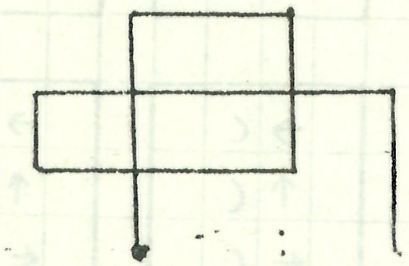
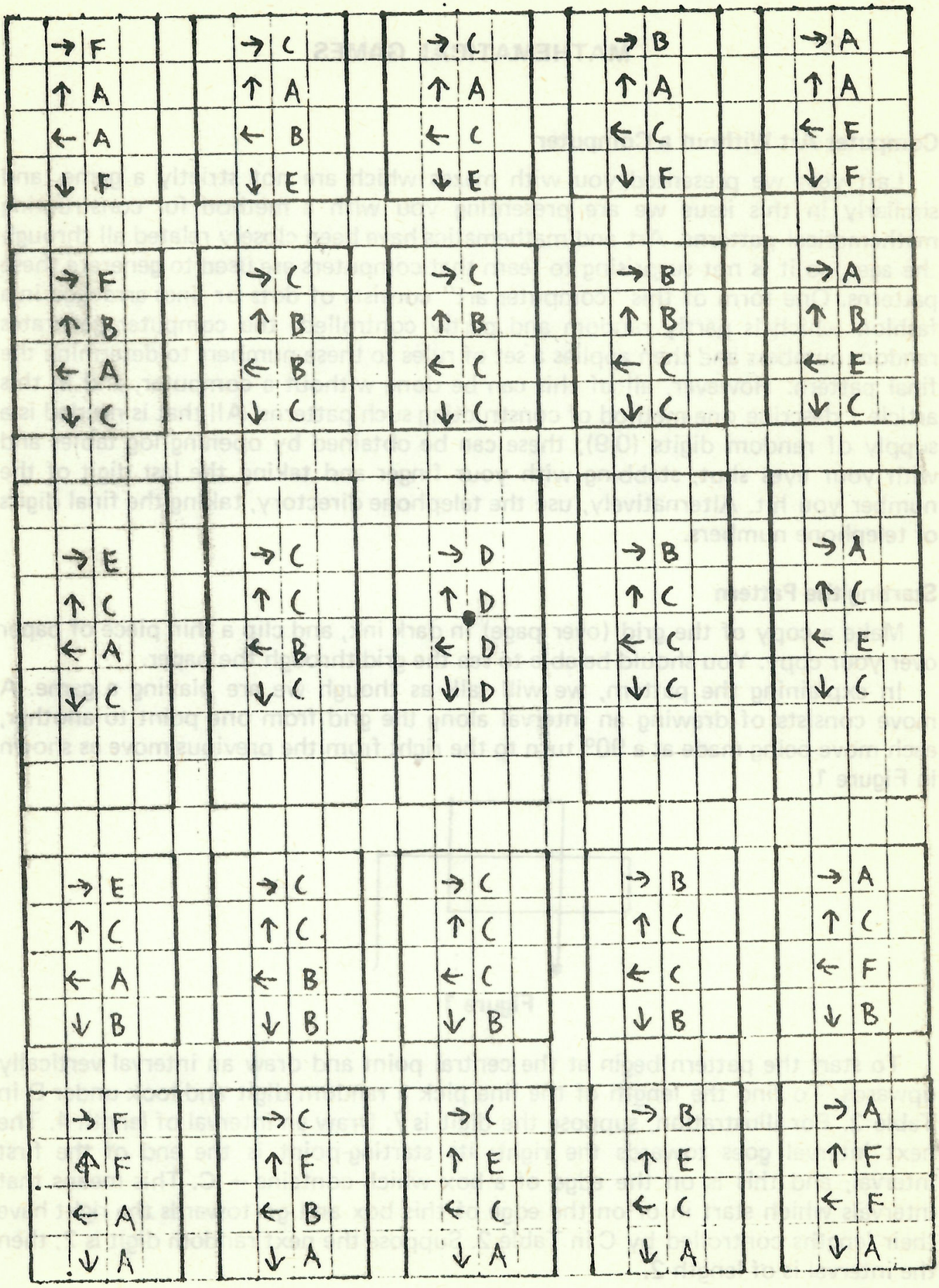


Figure 1

To start the pattern begin at the central point and draw an interval vertically upwards. To find the length of the line pick a random digit and look under D in Table 2. For illustration, suppose the digit is 7. Draw an interval of length 4. The next interval goes towards the right; its starting-point is the end of the first interval, and this is on the edge of a box which contains → C. This means that intervals which start in or on the edge of this box and go towards the right have their lengths controlled by C in Table 2. Suppose the next random digit is 2; then the interval is of length 2.


→ F
↑ A
← A
↓ F
→ C
↑ A
← B
↓ E
→ C
↑ A
← C
↓ E
→ B
↑ A
← C
↓ F
→ A
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→ D
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← D
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→ B
↑ C
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← E
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→ F
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← A
↓ A
→ C
↑ F
← B
↓ A
→ C
↑ E
← C
↓ A
→ B
↑ E
← C
↓ A
→ A
↑ F
← F
↓ A

Grid

The next interval is vertically downwards. We are still on the edge of the same box as before, so the ↓ C in this box means this interval also has its length controlled by C in Table 2. Proceed drawing intervals like this until they start to cross existing intervals. Then further rules apply, as explained below.

The Main Rules

A *legal* move ends on a vacant point or on the central point and does not overlap any already existing intervals. To find the length of a move, proceed as follows:

i) Choose a random digit, and find the length from Table 2 according to the box you are in on the grid and the direction of the move. If this gives a legal move, make it.

ii) If the original move – as given by rule (i) is not legal but does not overlap an existing interval, consider successively longer moves. If one of these is legal, make it.

iii) If (ii) fails or if the original move overlapped an existing interval, consider successively shorter moves until a legal one is found.

iv) The pattern finishes if no legal move can be found, or if an interval terminates at the central point.

TABLE 2

A	
DIGIT	LENGTH
any	1

B	
DIGIT	LENGTH
0, 1, 2, 3	1
4, 5, 6	2
7, 8, 9	3

C	
DIGIT	LENGTH
0, 1	1
2, 3	2
4, 5	3
6, 7	4
8, 9	5

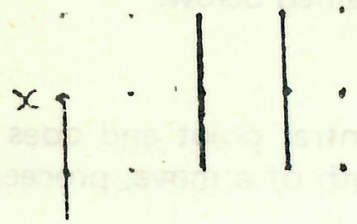
D	
DIGIT	LENGTH
0,1,2	1
3	2
4	3
5, 6, 7	4
8, 9	5

E	
DIGIT	LENGTH
0	1
1, 2	2
3, 4	3
5, 6, 7	4
8, 9	5

F	
DIGIT	LENGTH
0	2
1, 2, 3	3
4, 5, 6	4
7, 8, 9	5

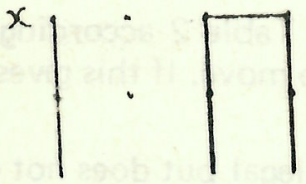
Examples: In all of these, we have just drawn an interval vertically upwards to the point x in the top left hand box of the grid, and so we have to draw an interval according to → F.

1)



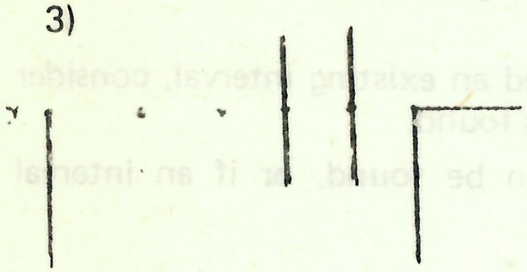
The random digit is 0, so rule (i) above gives a move of length 2. This is illegal, so we go to rule (ii). We consider first an interval of length 3: this is still illegal. But a move of length 4 is legal.

2)



The random digit is 8; so → F gives an interval of length 5. But this would overlap an existing interval. So we consider shorter intervals according to rule (iii) – the first legal move is of length 1, as 4, 3 and 2 are all illegal.

3)



The random digit is 2; → F gives a move of length 3. This is illegal, so we go to rule (ii). But length 4 is still illegal, and longer intervals run into an interval. So rule (ii) fails, and we try rule (iii). This gives a move of length 2, which is legal.

Before reading on, you should try your hand at a pattern following these rules.

Extra Rules

The above rules produce quite interesting patterns, but often the pattern gets trapped and finishes rapidly. This can be avoided by introducing extra rules, as follows. Call a move super-legal if it is legal, and it is possible to make five more legal moves after it. The rules are:

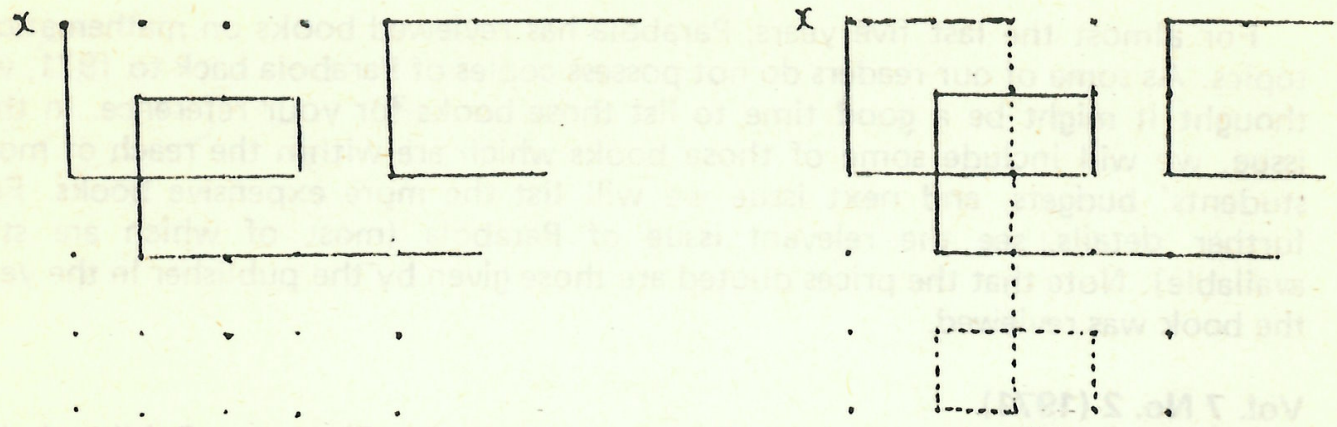
i') Use rules (i), (ii), (iii) as before, *except that only super-legal moves* are allowed. E.g. in rule (ii) consider longer and longer moves until a super-legal one is found.

ii') If there are no super-legal moves, make a move which gives the longest possible sequence of legal moves following it.

iii') A move which terminates in the central point is declared to be always super-legal.

iv') The pattern finishes if no legal move can be found or if a move terminates in the central point.

Example: Again we are at point x trying to go right.



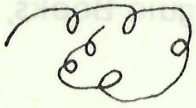
(a) (b)

A move of length 1 is legal, but not super-legal, as no further moves can be made after it. This is also true for length 3. The only super-legal move is of length 2. In (b) the super-legal move is shown dashed, and a sequence of five possible further legal moves is shown dotted. Note that I am not saying that the pattern will necessarily follow the dotted lines, just that the dashed line is super-legal because such a sequence of dotted lines exists.

With these extra rules, the pattern is much more likely to go for a long time before getting trapped. Note that legal moves can be very long; if you can see any sequence of five legal moves following a given move then the given move is super-legal, even if one of the legal moves is of length 10 or 15 or more.

Conclusion

The rather complicated collection of rules given above was my attempt to "mechanize" this sort of doodle:



You might like to modify the rules, or try to mechanize another sort of doodle, for example:

G.P. Monro

(Dr Monro is a lecturer at Sydney University. Please send us the results of your mathematical doodles and we will print the best in future issues of Parabola. Some examples by Dr Monro may be found scattered through this issue. — Editor)

Mastermind

Because we have not allowed for sufficient time for you to play this game, we are leaving its analysis until the next issue.