A Property of Some Recurring Decimals

Most people who are interested in mathematics are aware that the topic of recurring decimals has been, for a long time, a happy hunting-ground for mathematical oddities (this expression is meant to refer to the hunted, not the hunters). Perhaps, therefore, this rather elementary note need no very abject apology.

My attention was drawn to this particular "oddity" by noting the curious (at first sight) reciprocity between the pair of numbers, 27 and 37, which is seen on inspecting the two following true statements:

$$1/27 = 0.\dot{0}3\dot{7} \qquad (1)$$

and

$$1/37 = 0.\dot{0}2\dot{7}. \qquad (2)$$

Naturally enough, the first question which occurs to one's mind is, "Are there any more such number pairs?" The answer is, "Plenty!" However, some of these pairs need a little dressing up before the reciprocity shows up as clearly as it does in equations (1) and (2). The "dressing up" amounts to no more than using such facts as that 03 (i.e. no tens and three units) means just the same as 3 does and that, for example, $0.\dot{7}\dot{7}$ means just the same as $0.\dot{7}$ does.

If we write down the true statements –

$$1/3 = 0.\dot{3}$$

and

$$1/33 = 0.\dot{0}\dot{3},$$

the reciprocity observed in the case of 27 and 37 is vaguely suggested but by no means appears obvious. If, however, the first statement is re-written as

$$1/03 = 0.\dot{3}\dot{3}$$

the reciprocity immediately appears.

The following example calls for rather more dressing up.

$$1/99 = 0.\dot{0}\dot{1},$$

$$1/101 = 0.\dot{0}09\dot{9}.$$

If these statements are re-written as

$$1/0099 = 0.\dot{0}10\dot{1}$$

and

$$1/0101 = 0.\dot{0}09\dot{9},$$

the same type of reciprocity is again apparent.

The explanation of the relationships we have been looking at should be well within the powers of anyone who has an elementary knowledge of recurring decimals. Why not look into it?

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