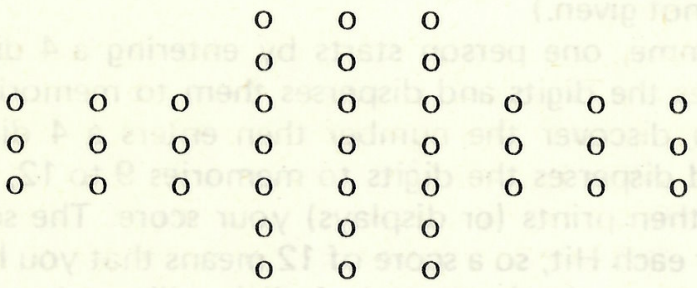
MATHEMATICAL GAMES

Solitaire

Solitaire is a game for one person. It is played with pegs or marbles on a board with holes in it, in the following pattern:



The game begins with every hole occupied except for the central hole. A move is made by jumping one peg over another next to it, and removing the peg over which it jumped. Thus for example, if the position were • • o, the peg on the left could jump over the middle peg which would then be removed, leaving the position o o •. Only horizontal and vertical jumps are allowed. The aim of the game is to end up with only one peg on the board (in the central hole). The game ends when no further move is possible.

You should devise your own notation for the game. If you succeed in ending up with just one peg in the centre, send us your solution. We will give $1 plus one year's free subscription to Parabola for each of the first ten correct solutions, and publish the best in the next issue.

Questions:

(1) What is the greatest number of moves that a game of solitaire can last?

(2) How many pegs can you put on the board so that no move is possible (other than 33 pegs which would clog the board)?

(3) What is the greatest number of pegs you can have on the board at the end of an actual game? (Again, a prize of $1 plus one year's free subscription to Parabola will be sent to anyone who sends a solution with more remaining pegs than any other reader.)

Computer Art

In our last issue, we described the "rules" for producing patterns, and invited our readers to send us their results, but no-one has sent any in yet. If you have a pattern, please send it so that we can print it in the next issue.

Mastermind

This game was described in Vol. II No. 3 where it was suggested that we might discuss the possibility of programming a calculator for playing the game. Since then, we have received three programmes: the first from Mr P. Sherley (a teacher at Riverstone High), the second from Mr P. Cox of Wahroonga and the third from Mr T. Croucher and Mr A. Smyth (teachers at Blakehurst High). Although the first two programmes were very good, the example from Blakehurst High, for the Canon Canola 1614P with Printer (optional) is felt to be the simplest and most compact and is printed below. (They also sent a programme which deals with repetitions but this is not given.)

To use the programme, one person starts by entering a 4 digit number. The machine then separates the digits and disperses them to memories 5, 6, 7 and 8. The person trying to discover the number then enters a 4 digit number. The machine separates and disperses the digits to memories 9 to 12, counts the Bulls, counts the Hits, and then prints (or displays) your score. The scoring system is: 10 for each Bull, 1 for each Hit, so a score of 12 means that you have 1 Bull and 2 Hits (and so one digit completely wrong). A digit will not be counted as a Bull and a Hit in any one guess. Thus the aim of the game is to score 40. We hope that you will enjoy playing Mastermind as much as we have at our school.

Procedure:

1. With machine ON, mode LRN, printer OFF enter the programme either manually or by punch cards.
2. With mode OPE
   (a) Press START
   (b) Enter 4 digit number
   (c) Press START
3. With printer ON
   (a) Enter Guessed number
   (b) Press START (Your guess will be printed). After a few seconds the machine gives your score.
   (c) If SCORE is less than 40, enter another guess. (If SCORE = 40 the machine returns to the beginning and waits for a new secret number.)

Notes:

1. If your school has no print attachment then replace '0' in step 98 with SJ, and omit step 99. When the machine stops and displays your score, then press START and it will continue.
2. Instructions SUJ, SFJ, and SRJ cannot be entered manually: they must be entered by a punch-card. They are instructions "108", "109" and "110".

Programme:

1	FJ
2	1
3	ENT
4	SM3
5	SUJ
6	CM1
7	SM8
8	SUJ
9	CM1
10	SM7
11	SUJ
12	CM1
13	SM6
14	RM3
15	SM5
16	CI
17	FJ
18	2
19	CM1
20	ENT
21	SM3
22	SUJ
23	CM1
24	SM12
25	SUJ
26	CM1
27	SM11
28	SUJ
29	CM1
30	SM10
31	RM3
32	SM9
33	RM12
34	RM8
35	SUJ
36	CM2
37	RM11
38	RM7
39	SUJ
40	CM2
41	RM10
42	RM6
43	SUJ
44	CM2
45	RM9
46	RM5
47	SUJ
48	CM2
49	RM12
50	RM7
51	SUJ
52	CM3
53	RM12
54	RM6
55	SUJ
56	CM3
57	RM12
58	RM5
59	SUJ
60	CM3
61	RM11
62	RM8
63	SUJ
64	CM3
65	RM11
66	RM6
67	SUJ
68	CM3
69	RM11
70	RM5
71	SUJ
72	CM3
73	RM10
74	RM8
75	SUJ
76	CM3
77	RM10
78	RM7
79	SUJ
80	CM3
81	RM10
82	RM5
83	SUJ
84	CM3
85	RM9
86	RM8
87	SUJ
88	CM3
89	RM9
90	RM7
91	SUJ
92	CM3
93	RM9
94	RM6
95	SUJ
96	CM3
97	RM1
98	◊
99	FD
100	4
101	0
102	≡
103	MJ
104	2
105	UJ
106	1
107	SFJ
108	CM1
109	RM3
110	SM4
111	→
112	SM3
113	x
114	1
115	0
116	≡
117	M4
118	RM4
119	SRJ
120	SFJ
121	CM2
122	≡
123	√
124	SC
125	MJ
126	3
127	1
128	0
129	M1
130	FJ
131	3
132	SRJ
133	SFJ
134	CM3
135	≡
136	√
137	SC
138	MJ
139	4
140	1
141	M1
142	FJ
143	4
144	SRJ

The strategy.

To our knowledge no mathematical analysis has yet been done to suggest the best strategy for Mastermind, so based on our experience of many games we would like to offer a strategy that we believe to be sound.

First guess:	Try any 4 digits.
Second guess:	Unless you had 4 Hits or better, try 4 different digits.
Third guess:	If you haven't had a total of 4 Hits (or better) then try the 2 remaining digits with 2 from previous guesses.
Fourth guess:	Based on the previous guesses determine the most likely result (that is the one that has most probability) and assume it to be correct.
Remaining guesses:	Here you have to play as it goes, but pursue your assumption until it is proved right or wrong. If you do this you will either have finished or discovered a new possibility to follow.

Finally, might we suggest that in most cases take the combination of digits that has the most possibilities. In most cases you should finish in 6 or maybe 7 guesses.

The above strategy was also submitted by Mr T. Croucher. After playing the game for a while, some of our readers might like to try to suggest a better strategy.

Answer to Graeme Elsworthy's polynomial trick:

Write $f(x_0)$ in base x_0 notation. Thus $f(x_0) = a_0 + a_1 x_0 + \ldots + a_n x^n{}_0$ where $a_n, a_{n-1}, \ldots, a_0$ are the "digits" of $f(x_0)$ to base x_0. The algorithm which allows you to do this gives unique values for $a_0, a_1, \ldots, a_n$ and so defines a unique polynomial $f(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$. For example, if you are told that $f(9) = 341$ then you write 341 (base 10) = 418 (base 9) and so the polynomial is $f(x) = 4x^2 + x + 8$.