A FORMULA FOR PRIME NUMBERS?

A number is prime if it has exactly two factors – itself and unity (one). Thus 2, 3, 5, 7 and 11 are the first five primes. Note that unity itself is not considered a prime number as it has only one factor – itself.

There are formulas for prime numbers, but they are not very useful. For example, the formula $n^0 + 2$ will always give a prime answer for any value of n, as the only possible value for n^0 is 1 and thus $n^0 + 2$ can only equal 3. Such a formula undoubtedly works, but could hardly be regarded as an important mathematical breakthrough. There are two easily provable statements which help us in trying to formulate a formula for prime numbers. These statements are:

(1) A number is prime if it has no factors less than its square root, other than unity.

(2) A prime number p does not divide evenly into another number A–B if it divides into A and not B, or vice versa.

As I have said, these statements are easily proven. To do so we simply assume the opposite.

If (1) were false, it would mean there was a number N with two prime factors p and q such that $pq = N$, $p > \sqrt{N}$, $q > \sqrt{N}$. As both p and q are greater than $\sqrt{N}$, then $pq > \sqrt{N} \times \sqrt{N}$, and so $N > N$. This is clearly a contradiction.

If (2) were false, then the number A–B could be factorised as $p(A/p - B/p)$ with both factors being integral. As either A or B, but not both, is divisible by p, then either A/p or B/p must be integral, and the other one non-integral. Thus $A/p - B/p$ would be the difference of an integer and a non-integer, and $A/p - B/p$ must be non-integral. It follows that in such circumstances p does not divide evenly into A–B.

So, with both statements proven, we will try to use them successfully. All that would be needed for a prime number formula would be a formula whose values satisfied statement (1) above. But how does one determine if an arbitrarily selected number N is divisible by numbers less than its square root? By statement (2) of course!

By statement (2), it should be clear that if we made $N = A - B$, N would not be divisible by any factors of A which are not also factors of B. So what if we made A and B have no factors in common other than unity? This would mean that A–B, and consequently N, would have no factors in common with A or B, other than unity.

Thus, if A and B between them have as factors all the prime numbers up to and including the n'th prime, and their only common factor is unity, then A–B would not be divisible by any number up to and including the n'th prime.

As any prime factor which divides into A or B must divide into AxB (and vice versa), the statement "A and B between them have as factors all the prime numbers up to and including the n'th prime" is equivalent to the more concise one "AxB is divisible by all primes up to and including the n'th prime". If we add to this the fact that the n'th prime number is greater than $\sqrt{(A-B)}$, then A–B must have no factors less than its square root, other than unity. Such a number is not necessarily prime – it just has no factors other than itself and unity. But unity might equal the number A–B, and as explained earlier, unity is not a prime number. This however is easily rectified. We shall change "$\sqrt{(A-B)}$ < the n'th prime number" in our assumption to "$1 < \sqrt{(A-B)}$ < the n'th prime number". This, however, means that if $n = 1$, the n'th prime number is 2, and there is no integer between 1 and 2. Therefore $n \neq 1$. n must also be positive, so we must say that $n > 1$. Thus our theorem reads:

Let n be any integer greater than 1. Then A–B is prime where

(1) The only common factor of A and B is unity;

(2) AxB is divisible by all prime numbers up to and including the n'th prime; and

(3) $1 < \sqrt{(A-B)}$ < the n'th prime number.

(B is obviously not 0 as, if it were, A would divide B.)

Substituting values for n we get

n = 2: AxB is divisible by 2x3 = 6. A possible solution is:

A = 2x5 = 10
B = 3
A–B = 7, which is prime

n = 3: AxB is divisible by 2x3x5 = 30. A possible solution is:

A = 2x3x5 = 30
B = 7, 11, 13, 17, 19, or 23
A–B = 23, 19, 17, 13, 11, or 7, which are all prime

n = 4: AxB is divisible by 2x3x5x7 = 210. A possible solution is:

A = 3x3x5 = 45
B = 2x7 = 14
A–B = 31, which is prime, etc.

Note. It is also curious to note that where I replaced n with 3, a curious pattern developed between the value of B and that of A–B. Thus, 7 + 23 = 30, 11 + 19 = 30 etc. Such a pattern might occur with 210 (2 x 3 x 5 x 7), 2310 (2 x 3 x 5 x 7 x 11) etc. Maybe someone would care to investigate this.

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[This is a good article by Philip. It raises another question which someone might like to try: Is it always possible to choose A and B so as to satisfy (1), (2) and (3) of Philip's theorem? I have so far done it for n = 2,3,4,5 and 6: how far can you get? – Editor]

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RESEARCH CORNER

Palindromic Numbers

A palindrome is usually thought of as a word or a sentence which reads the same backwards or forwards. Some examples of palindromic words are NOON, LEVEL and TUMUT, and a famous example of a palindromic sentence is the one imputed to Napoleon: ABLE WAS I ERE I SAW ELBA. We can also call positive integers palindromes when they read the same backwards as forwards. Some examples of these are: 525, 11, 1771 and 12321.

Another example may be obtained as follows:

Write down a number	47	
Reverse it	74	
Add	121	— a palindromic number.

Let us try that again:

Write down a number	67	
Reverse it	76	
Add	143	— no luck!
Reverse it	341	
Add	484	— a palindromic number.

Now it is over to you: Choose any positive integer, reverse it and add the two numbers, repeat this process and continue until you get a palindromic number. Do you always get a palindromic number after a finite number of steps? All attempts or suggestions will be accepted and the best printed in Parabola during 1977 – there may even be a prize! (I have already received some attempts from Blakehurst High.)

If you cannot get anywhere with numbers written to base 10, you may like to try other bases. For example in base 5 we have

$$\begin{array}{r} 24 \\ \underline{42} \\ 121 \end{array}$$