YOUR LETTERS

Dear Sir,

I was recently reading your article on Magic Squares (Vol. 12 No. 1) and started doing a few.

I did 3 to 9 and then started on 10 but soon discovered that I could not adapt the 6 unit Magic Square to the 10. Could you please print the 10 so that I may see how it is done.

As well as this I have a code. Each column is a word and each 4 digit number is a letter (Hint: know your Logarithms.)

6462	2574	5598
8036	6653	3632
4489		9120
1501		6942
3464		
5465		
3824		
8964		

David Budd
Bass High

[See page 17 for David's answer and the magic square. – Editor]

Dear Sir,

Reading your reply to my letter on a "formula for prime numbers" I have taken the case where p,q are primes and q = 6 + p.

To do this I have used the two theorems about the prime q which you gave in Vol. 12 No. 2.

$$(q-1)(q-2)(q-3)(q-4)(q-5)p! = (q-1)! \equiv -1 \pmod{q}$$

So $$32(q-1)(q-2)(q-3)(q-4)(q-5)p! \equiv -32 \pmod{q} \qquad (1)$$

$$32 \times 2^p = 32 \times 2^{q-6} = 2^{q-1} \equiv 1 \pmod q$$

So $32(q-1)(q-2)(q-3)(q-4)(q-5)2^p \equiv$

$$\equiv q^5 - 15q^4 + 85q^3 - 225q^2 + 274q - 120 \pmod q$$

$$\equiv -120 \pmod q \qquad (2)$$

Adding (1) and (2), we have

$$32(q-1)(q-2)(q-3)(q-4)(q-5)(p! + 2^p) \equiv -152 \pmod q$$

Since $32(q-1)(q-2)(q-3)(q-4)(q-5) \equiv -120 \times 32 \pmod q$

$$\equiv -3840 \pmod q,$$

we must have $152 \equiv 3840 \pmod q$

or $q | 3688 = 2^3 \times 461$

Thus $q = 461$ and $p = 455$, which is composite.

Hence it is impossible to find prime numbers p and $q = p + 6$ such that q divides $2^p + p! - 1$.

Danny Zulaikha.

[Has anyone been able to find any prime p (other than 13) for which $p! + 2^p - 1$ is composite? – Editor]

Dear Sir,

When I was at school I appreciated the fact that a university department had so much interest over school kids, especially in mathematics which I was most interested in.

Being an avid reader of your magazine, even though I stopped contributing three years ago when I left school, I read with dismay that you even thought of ceasing publication due to such a small readership.

Unfortunately, unless you continue in mathematics when you enter university, you find yourself quite quickly breaking any relationships and losing interest in it, a fact which I have had many regrets about. The only continuing contact has essentially been through Parabola.

I mention this for two reasons. Firstly, I, and in fact many maths-mad school leavers would be most disappointed if you ceased publication. Secondly, it might be an idea if you extended your readership to school leavers (in fact I'm sure a large no. of your readers are non school students) with certain restrictions ensuring that the standard never exceeds that of school standard, and that school mathematicians are always given first considerations.

I hope to keep receiving Parabola for a long time.

Jerry Schwartz

[Parabola is supposed to be for anyone interested in Mathematics – especially

school students. But its existence depends on your interest and we can only know your views if you write to us. We would love any contribution anybody can offer – especially you. – Editor]

Dear Sir,

I would like to submit, for proof and verification, an iterative formula for root approximations for functions of the form $ax^n + b$. I realise, also, its application to finding roots of different magnitude. The extent to which I have tested it reveals its validity in finding fractional powers, integral and negative powers. I submit it for further testing and a hopeful proof by induction or otherwise.

The formula is:

$$z_{n+1} = [a + (c-1)z_n^{\,c}]/cz_n^{\,c-1}$$

where the numbers are of the form $z_n \cong \sqrt[c]{a}$. z_1 is the first approximation and substitution of this into the formula gives z_2 which, with further substitution, gives z_3 and so on.

Paul Hogwood

[Editor's reply: Your iterative formula for finding roots of $z^c = a$ is in fact a good one. It is an example of a method for approximating roots of any equation of the form $z = f(z)$ where the derivative of the function $f(z)$ is 0 at the root. The iterative formula for solving this is to choose a value z_1 close to the root and then to put $z_{n+1} = f(z_n)$ for $n = 1,2,3, \ldots$ The value of z_n then converges to the required root.

In your example, we wish to find a function $f(z)$ satisfying the above conditions such that "$f(z) = z$" is equivalent to "$z^c = a$". Now "$z^c = a$" is equivalent to "$z = a/z^{c-1}$" which does not satisfy our condition. Writing $(1+\alpha)z = z + \alpha z = (a/z^{c-1}) + \alpha z = (1+\alpha)f(z)$, we have

$$(1+\alpha)f'(z) = -[(c-1)a/z^c] + \alpha.$$

At the root, $z^c = a$ and so

$$0 = (1+\alpha)f'(z) = -(c-1) + \alpha.$$

Thus $1 + \alpha = c$, giving

$$z = f(z) = [a/z^{c-1} + \alpha z]/(1+\alpha) = [a + (c-1)z^c]/cz^{c-1}$$

which yields your iterative formula.]

Dear Sir,

Regarding Question 2 in the Senior Division of the Mathematics Competition, 1976, I happened to notice that another sequence of six prime numbers in Arithmetic Sequence can be found by another method. The numbers given were: 5, 11, 17, 23, 29. I noticed that if you tacked a '3' onto the numbers given, you obtain another sequence of numbers which are prime and in Arithmetic Sequence, with a common difference of 60. Therefore the sixth prime number in the sequence becomes 353 (293 + 60). Therefore the sequence becomes:

53, 113, 173, 233, 293, 353.

Which, are six prime numbers in Arithmetic Sequence.

N.G. Serafim, Cranbrook School

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Answer to David Budd's code (see "Your Letters"): PARABOLA IS GOOD. Method: Put a decimal point in front of each number then look it up in the Table of Four-figure Antilogarithms. Write down the answer and take the last 2 digits from the first 2 digits of what you wrote down. The answer will be the letter corresponding to that number (where a corresponds to 1, b to 2, . . . , z to 26).

92	99	1	8	15	67	74	51	58	40
98	80	7	14	16	73	55	57	64	41
4	81	88	20	22	54	56	63	70	47
85	87	19	21	3	60	62	69	71	28
86	93	25	2	9	61	68	75	52	34
17	24	76	83	90	42	49	26	33	65
23	5	82	89	91	48	30	32	39	66
79	6	13	95	97	29	31	38	45	72
10	12	94	96	78	35	37	44	46	53
11	18	100	77	84	36	43	50	27	59

A magic square of order 10 (see Vol. 12 No. 1)