SCHOOL MATHEMATICS COMPETITION 1976
EXAMINERS' REPORT

Your results were, I think, better this year than last. There was a greater willingness to tackle all the questions. The largest number of correct answers was for Question 1 and the next largest was for Question 2.

Nobody got the first part of Question 5, though there were some valiant attempts. Some of you asserted that if the first part were true, the second part couldn't be, which is a bit like saying that if all the students in a class passed an exam with pass mark 50, none could have got 50!. I wasn't surprised – I found it difficult and when you come to the question towards 4.30 or so, it's not ideal conditions for problem solving.

Full marks were obtained for the remainder of the questions by various of you though no single individual got full marks for all 4. It would seem to me that the paper was more attractive to you this year; one student, I forget which, said that she/he liked it.

Brief comments.

Question 1: Most of the marks were for the first part.

Question 2: Arithmetic errors were almost excused, markwise.

Question 3: Probably the fewest marks were awarded for this question, though some got the right idea without carrying it through to the end. (They were rewarded.)

A few read (a + 2b)/(a + b) as (a + 2b) x (a + b) instead of the fraction $\frac{(a+2b)}{(a+b)}$. [We'll try to avoid this in future.]

Question 4: Pt 2 was done correctly by quite a few competitors.

Question 5: Again people scored quite well on Pt 2. There are plenty of other solutions apart from the one given in Parabola.

I would like to try to help you in future competitions, while your entries are still fresh in my mind.

(1) **Neatness.** Some competitors indicated that they were very worried about this. Well, I don't mark for neatness of writing, although when I get an answer that's quite impossible to read, I give it zero; so would you. There's no need to

rewrite your answers, even if they do contain mistakes, unless you've got time to do so and want to do so.

(2) **Clarity.** This is quite different from neatness – except that it indicates neatness of thought. I can't read what's in your mind – only what you put down on the paper. Some of the explanations given in the question about the knights, the wines and King Arthur may have cheated their writers out of the marks their thoughts should have entitled them to. The key point in Part 1 was to prove that each 3rd knight *had* to have wine of the same colour while a solution to Part 2 was to group 2 reds and a white together all the way round. Both answers involved the number 3. It wasn't always clear to me in Part 1 whether you understood the key point, even if you hadn't given a fully satisfactory proof, or whether you were, in effect, doing Part 2 twice. Say what you mean!

(3) **Problem solving.** I am hesitant about advising on this; this is usually a very individual matter. Because you set about it one way and I in another, it certainly doesn't make my technique better than yours. In fact, every year, I find some competitor has done a problem in an ingenious way I'd not thought of, and it's a pleasure to see it. But there are a few tips I could give to newer competitors.

These are *thinking* questions, not just mechanical ones. So, first of all, think about a question before you start on it. Of course, you may well want to think with your pen in hand on the unlined sheets of the books. There's nearly always a pattern, a principle, to seek before you start – which should lead to a reasonably short proof. Mind you, as examiner, I'm anxious to see your thoughts; I can usually pick them up from your rough working so don't cross it out – in fact, don't cross out anything if you can help it, or cross out with one line only. (Sometimes your "mistake" is right and your corrected version wrong, and I want to give you marks.)

I think our questions can usually be divided into two kinds; those like Question 1 Pt 1, Question 3, Question 4 Pt 1 and Question 5 Pt 1, where you are asked to prove something true for *all* cases and the others where you have either to get the unique answer as in Question 2 or else have to give *an* answer, where there well may be more than one, as in Question 1 Pt 2 and Question 4 Pt 2 and Question 5 Pt 2. In general, the second kind are easier to do (though less interesting) than the first kind. You can put your answer down and check it to see that it works.

Let me discuss Question 3 as an example of the first kind of question. You have to prove something true for *all* positive integers a and b. You're a dead optimist – and a dead duck as far as marks go – if you prove it for say, a = 1, b = 1 and then a = 1, b = 3 etc. There are an infinite number of positive integers – wherever you stop counting there's always another to come – so you can't possibly prove the statement true for *all* positive integers that way. That means that the pronumerals a and b must appear all the way through your proof, as you can.

It is usually easier to work with rational numbers rather than surds so I give the proof I prefer and which the 1st Prizewinner and one or two others gave.

There are 3 possibilities (i) $a/b = \sqrt{2}$, (ii) $a/b < \sqrt{2}$, (iii) $a/b > \sqrt{2}$.

(i) This is impossible by the ancient Greek proof usually given in High School. If you don't know the proof, ask your teacher to show you.

(ii) This means that $a^2 < 2b^2$

We have to prove $\sqrt{2} < (a+2b)/(a+b)$ or $2 < (a+2b)^2/(a+b)^2$

This will be true if $2(a+b)^2 < (a+2b)^2$

i.e. if $2(a^2+2ab+b^2) < a^2+4ab+4b^2$

or $2a^2+4ab+2b^2 < a^2+4ab+4b^2$

or $a^2 < 2b^2$

But the last statement is true and therefore $\sqrt{2} < (a+2b)/(a+b)$

(iii) The proof here is the same as in (ii) with < changed throughout to >. The second part is now straight forward; after all the method that worked for 2 may well work for n. It does, with $a^2 < nb^2$. We try

$$n(a+b)^2 < (a+nb)^2$$

which gives $(n-1)a^2 < (n^2-n)b^2$

or $a^2 < nb^2 \quad (n > 1)$ etc.

May I finish by recommending strongly to you the paper back "How to solve it" by Professor G. Polya. You will find it very helpful and very readable. It is a Doubleday Anchor Book.

M.G. Greening

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Sonnet

If I should die think only this of me:
I entered in the competition here
(Although I think that 2 and 2 is 3,
And of quadratic 'quations go in fear)
I braved the journey out to Kensington
On foot, by train and finally by bus
In hopes that I would have a lot of fun
And not get worried or get very frus
trated by the questions (which were hard)
And I confess I've not been sad at all
Despite the questions which I tried in vain
I now shall have to leave my role as bard
For I must go – Although I've had a ball
But don't despair, I shall be back again.

The above sonnet was written by Louise Hart during the School Mathematics Competition. Do we have any more mathematical poets amongst our readers?