PROBLEM SECTION

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions, and the names of those who submit solutions before the publication of the next issue, will be published in Vol. 13 No. 1.

Although the problems are unclassified, the first 4 problems should require no more mathematical knowledge than is learnt in first or second form and the next 4 problems should require no more mathematical knowledge than is learnt in the first 4 forms of High School. Senior students are thus encouraged to attempt the later problems.

Note: All of the following problems are taken from the Wisconsin mathematical, engineering and scientific talent search.

321. The following factorisations of numbers are true:

$$12 = 3.4;\ 1122 = 34.33;\ 111222 = 334.333;\ 1111222 = 3334.3333$$

Can this scheme be continued indefinitely? Prove your answer.

322. Suppose there were 250,000 people in Sydney in 1968 who made between \$8,000.00 and \$9,000.00. Show there were at least 3 people who made the same salary down to the last cent.

323. Twenty-six entrants, with names A, B, C, . . . , Z, play a chess tournament, each against all others. Score 2 points for a win, 1 for a draw, and 0 for a loss. No one's total was odd, there were no ties, and they ended in the order A, B, C, . . . , Z. What was the result of the match between M and N? Prove it.

324. Suppose that five points are located in a square of side length 1. Prove that at least two of the points must be within $\sqrt{2}/2$ of one another.

325. The sides of a triangle are a, b, c units where a, b, c are integers and $a \leqslant b \leqslant c$. If c is given, show that the number of different triangles is $\frac{1}{4}(c+1)^2$ or $\frac{1}{4}c(c+2)$ according as c is odd or even.

326. Suppose that mn boys are standing in a rectangular formation of m rows and n columns. Suppose that the boys in each row get shorter going from left to right. Suppose someone rearranges each column, independently of one another, so that going from front to back the boys get shorter. Show that the boys in each row still get shorter going left to right.

327. If a pack of playing cards is shuffled systematically and the operation of shuffling repeated exactly, then after a certain number of repetitions of the operation, the original order of the pack will be restored. Suppose the pack is shuffled as follows: Hold the pack face down in the left hand; in the right hand, take the top half of the pack and insert it into the lower half so that each right-hand card is above the corresponding left-hand card.

(a) After how many shuffles is a 52-card pack returned to order?

(b) After how many shuffles is a 26-card pack returned to order?

328. Six circular areas are lying in the plane so that no one of them covers the center of another. Show that there is no point in common to all six circular areas.

329. Consider the following array of natural numbers similar to Pascal's triangle. If we denote the nth row of the triangle by $a_{n1}, a_{n2}, a_{n3}, \ldots, a_{n,n-1}, a_{nn}$, then the law of formation is given by

$$a_{n1} = a_{nn} = 1$$

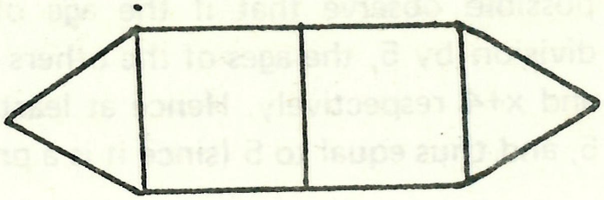
and for $2 \leqslant i \leqslant n-1$

$$a_{n,i} = (n-i+1)a_{n-1,i-1} + ia_{n-1,i}.$$

n					
1	1				
2	1	1			
3	1	4	1		
4	1	11	11	1	
5	1	26	66	26	1

For example, $a_{5,2} = (5-2+1)a_{4,1} + a_{4,2} = 4.1 + 2.11 = 26$, and $a_{5,3} = (5-3+1)a_{4,2} + 3a_{4,3} = 3.11 + 3.11 = 66$. (You should construct rows 6, 7, and 8 to understand the method of construction.) Find a simple formula involving n, for the sum s_n of the nth row, $s_n = a_{n1} + a_{n2} + a_{n3} + \ldots + a_{nn}$ and prove it. ($s_1 = 1$, $s_2 = 2$, $s_3 = 6$, ...).

330. Certain convex polygons can be dissected into squares and equilateral triangles all having the same length of side. For example, the illustration shows a hexagon dissected in such a way. If a convex polygon can be dissected in this way, how many sides did it have originally? Prove your answer.



331. Suppose that n^2+1 boys are lined up shoulder-to-shoulder in a straight line. Show that it is always possible to select $n+1$ boys to take one pace forward so that going from left to right their heights are either increasing or decreasing.

332. In a number of years equal to the number of times a pig's mother is as old as the pig, the pig's father will be as many times as old as the pig as the pig is years old now. The pig's mother is twice as old as the pig will be when the pig's father is twice as old as the pig will be when the pig's mother is less by the difference in ages between the father and the mother than three times as old as the pig will be when the pig's father is one year less than twelve times as old as the pig is when the pig's mother is eight times the age of the pig.

When the pig is as old as the pig's mother will be when the difference in ages between the pig's father and the pig is less than the age of the pig's mother by twice the difference in ages between the pig's father and the pig's mother, the pig's mother will be five times as old as the pig will be when the pig's father is one year more than ten times as old as the pig is when the pig is less by four years than one-seventh of the combined ages of his father and mother. FIND THEIR RESPECTIVE AGES. (For the purposes of this problem, the pig may be considered to be immortal.)

Solutions to Problems 309–320 (Vol. 12 No.2)

309. In a family with 6 children, the five elder children are respectively 2, 6, 8, 12 and 14 years older than the youngest. The age of each is a prime number of years. How old are they? Show that their ages will never again all be prime numbers (even if they live indefinitely).