THE 1976 SUMMER SCIENCE SCHOOL

TESSELLATIONS WITH CONVEX EQUILATERAL PENTAGONS

In 1976 for just the second time, the University of New South Wales held a Summer Science School, and for the first time Mathematics was involved.

Forty schools across New South Wales were asked to nominate their two best Science/Mathematics students in Year 11, and they were invited to take part. The students were given a choice of sixteen projects (seven in Chemistry, seven in Physics, two in Mathematics) and were divided into sixteen groups of about five each. They spent a week in late November at the University, doing research under the supervision of a member of staff. At the end of the week, parents and teachers were invited to a session in which students presented their findings.

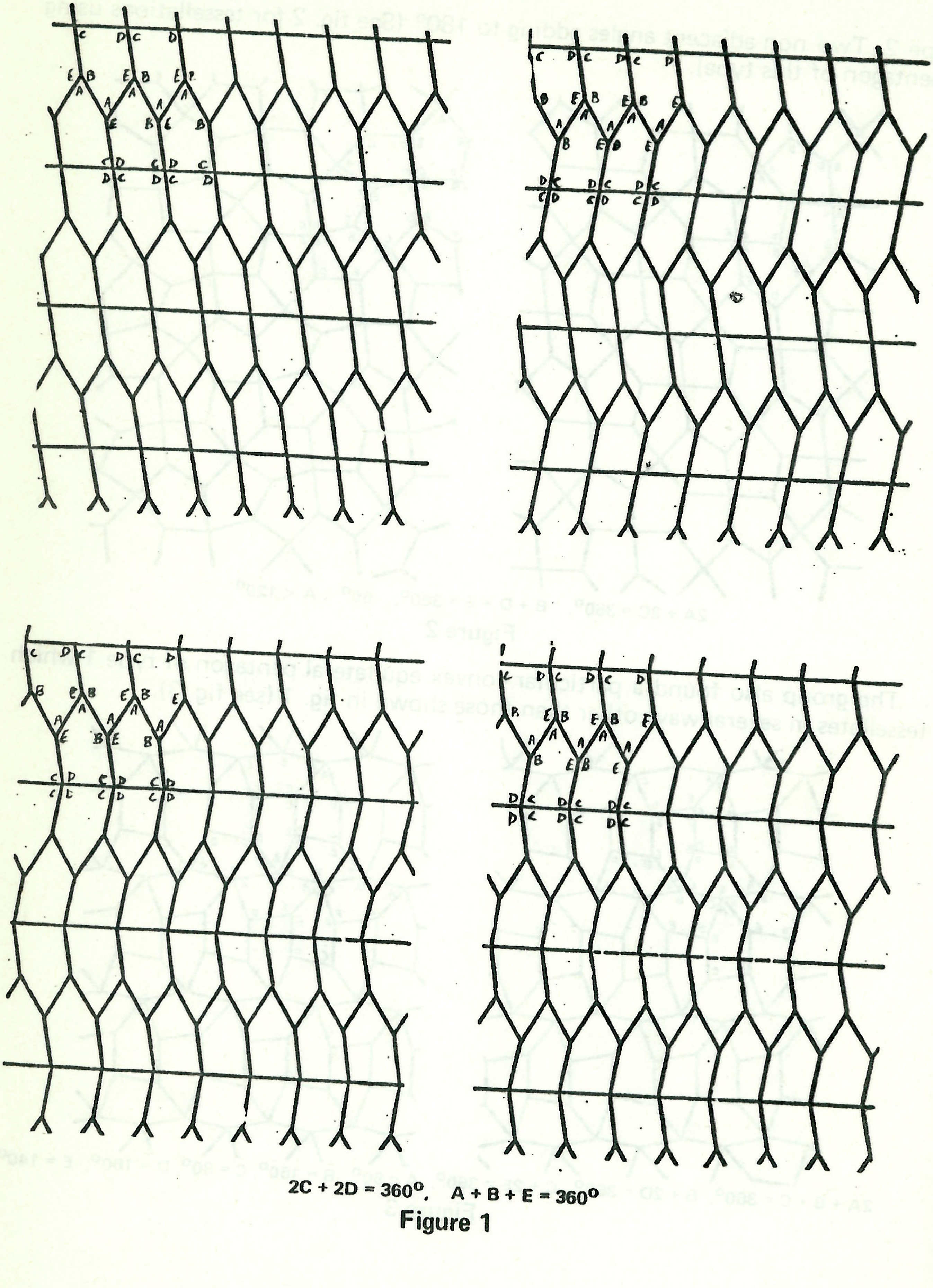
It is expected that this Summer Science School will be held each year, and, if anything, it will grow. I hope we see *you* at a future Summer School.

The two mathematical projects were "Intersection Properties of Subsets" supervised by Dr D.C. Hunt, and "Tessellations of the Plane by Pentagons" supervised by Prof. G. Szekeres.

I had a little to do with the second of these projects, and this is what I want to tell you about. The students involved were: Alan Fekete (Sydney Grammar), Louise Hart (Abbotsleigh), Sue Spencer (Pittwater High), Jenny Blythe (St George Girls), Michael Birrell (Macarthy High) and Rodney Burke (Tamworth High). They spent their time on such varied activities as listening to lectures, reading, cutting and pasting, and using a computer.

Imagine you have an infinite supply of identical jigsaw pieces. If these can be placed so as to cover the entire plane without gaps or overlaps then the piece is said to tessellate, and the pattern in which they are arranged is called a tessellation. The object of the week's activity was to find all convex equilateral pentagons which tessellate. (A polygon is called equilateral if all its edges have the same length, and is called convex if all its angles are less than 180°.) The outcome of the investigation was that the group found two types of convex equilateral pentagons which tessellate.

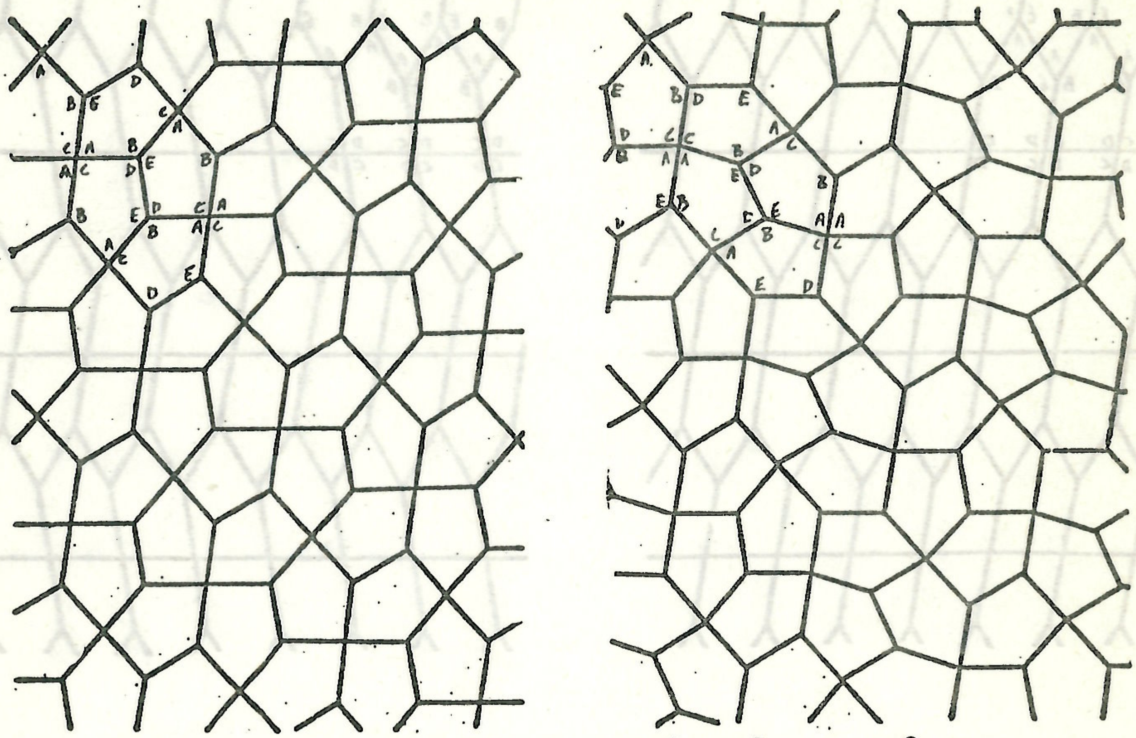
Type 1. Two adjacent angles adding to 180° (See fig. 1 for tessellations using a pentagon of this type).



$2C + 2D = 360^o, \quad A + B + E = 360^o$

Figure 1

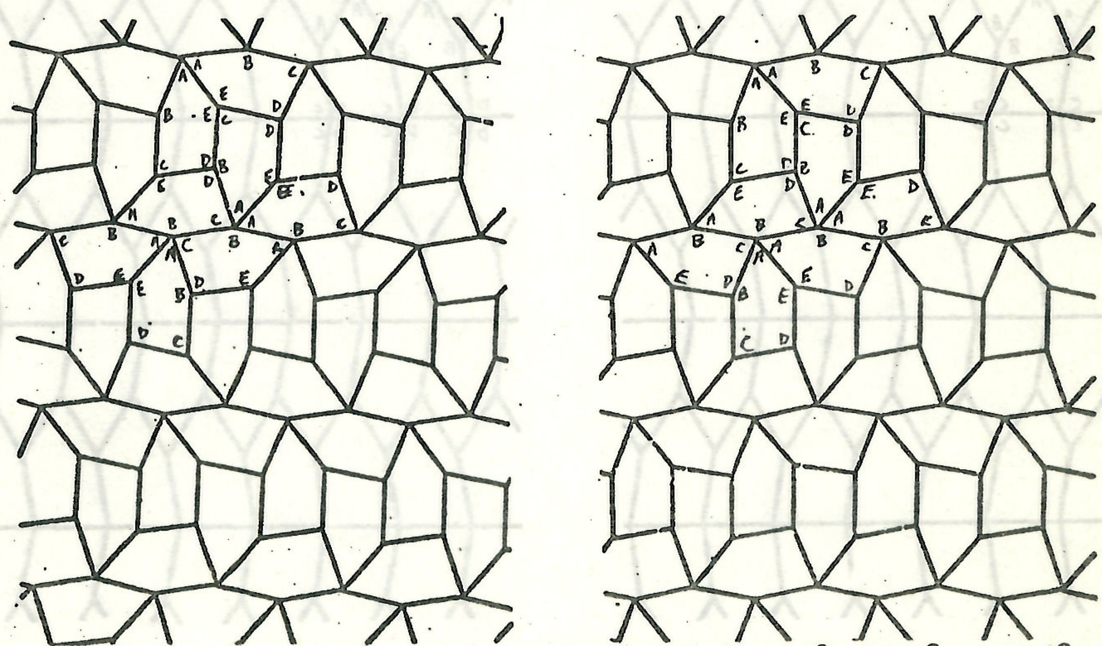
Type 2. Two non-adjacent angles adding to 180° (See fig. 2 for tessellations using a pentagon of this type).



$2A + 2C = 360^{\circ}$, $B + D + E = 360^{\circ}$, $60^{\circ} < A < 120^{\circ}$

Figure 2

The group also found a particular convex equilateral pentagon of type 1 which tessellates in several ways other than those shown in fig. 1 (see fig. 3).



$2A + B + C = 360^{\circ}$, $B + 2D = 360^{\circ}$, $C + 2E = 360^{\circ}$, $A = 60^{\circ}$, $B = 160^{\circ}$, $C = 80^{\circ}$, $D = 100^{\circ}$, $E = 140^{\circ}$

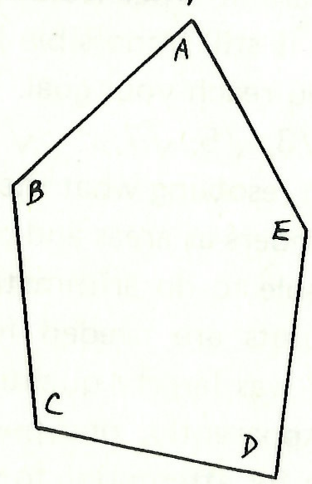
Figure 3

The story may well have ended here, except for the chance discovery I made of the tessellation shown on page 18 (using the same tile as in fig. 3) remarkable for being a non-periodic tessellation. Professor Szekeres communicated this tessellation to Mr Martin Gardner, writer of the "Mathematical Games" section of the magazine "Scientific American", and to cut a long story short, it turns out that a Californian, Mrs Marjorie Rice, has discovered several tessellations that we missed. The figure on page 23 shows a tessellation using, yet again, the tile in figure 3. The left-hand figure on page 36 shows a tessellation using a particular tile of type 2, and the right-hand figure on the same page shows a tessellation using a tile of neither type 1 nor type 2.

Whether or not we now have a list of all tessellations using convex equilateral pentagons remains an open question. I shall certainly work on this problem, and may report on it in a later issue. Meanwhile, if you have any comments or queries, let us have them.

Problems

During the Summer School, A. Fekete proved that, in a convex equilateral pentagon with angles A, B, C, D, E as shown,



the following relations hold:

$$\cos A = \cos C + \cos D - \cos(C + D) - \tfrac{1}{2}$$
$$\cos B = \cos D + \cos E - \cos(D + E) - \tfrac{1}{2}$$
$$\cos C = \cos E + \cos A - \cos(E + A) - \tfrac{1}{2}$$
$$\cos D = \cos A + \cos B - \cos(A + B) - \tfrac{1}{2}$$
$$\cos E = \cos B + \cos C - \cos(B + C) - \tfrac{1}{2}$$

1. Use these relations to find the angles in the particular convex equilateral pentagon in fig. 2, which is symmetric.
2. Prove the above relations.

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