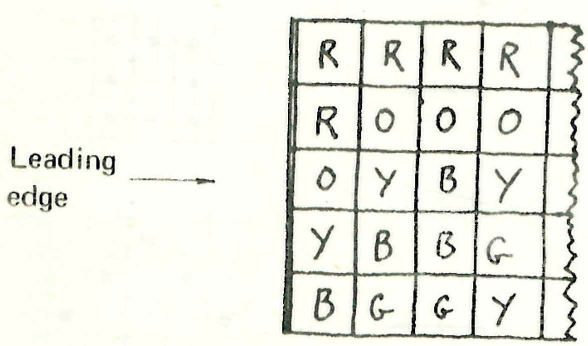
MATHEMATICAL GAMES

TAKE – ALL

Take – all is a game for two people. It is played on a board consisting of 5 rows of holes with 8 holes in each row and forty marbles, eight marbles of each of five different colours.

To begin the game, the forty marbles are scattered randomly into the holes. The players then take turns to remove marbles from the board according to the rules, the aim being to end up with more marbles than one's opponent after all the marbles have been removed from the board.

At each turn, the player whose turn it is chooses one of the five colours. He then removes at least one, and as many as he chooses, of the "exposed" marbles of that colour. A marble is said to be "exposed" if it is the closest marble in its row to the leading edge, which is defined to be the edge on the first player's left hand side. Note that as a player removes marbles from the board he may cause other marbles of the same colour to become exposed. He may remove these in the same turn. For example, in the situation below, the first player may remove some or all the red marbles (R) shown, but he may remove none of the green marbles (G) and may only remove one marble of any of the other colours shown. If he removes the red marble in the second row he allows the second player to remove all the orange marbles (O) shown.



To play Take – all well, you need a lot of foresight. Have fun!

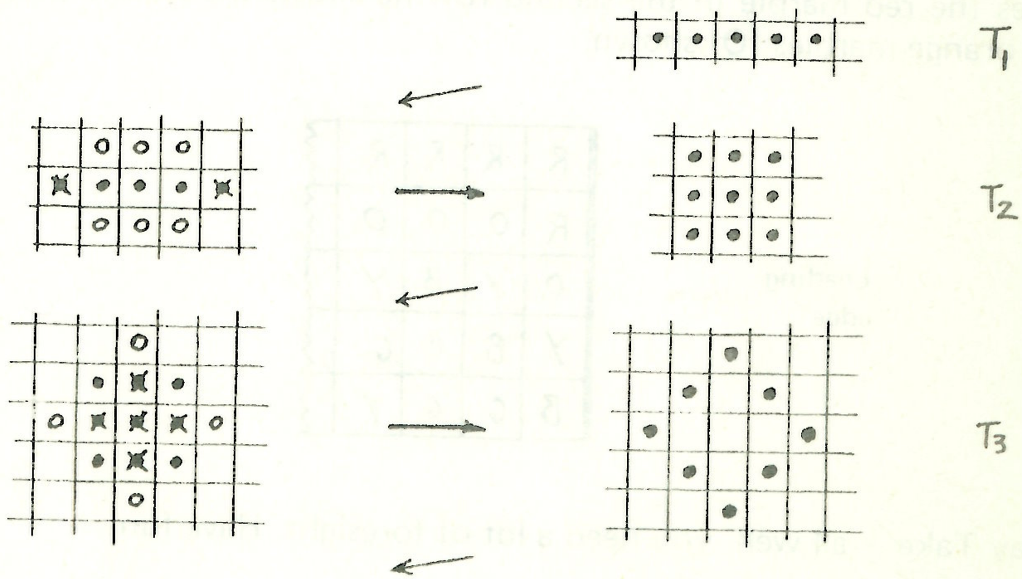
Questions

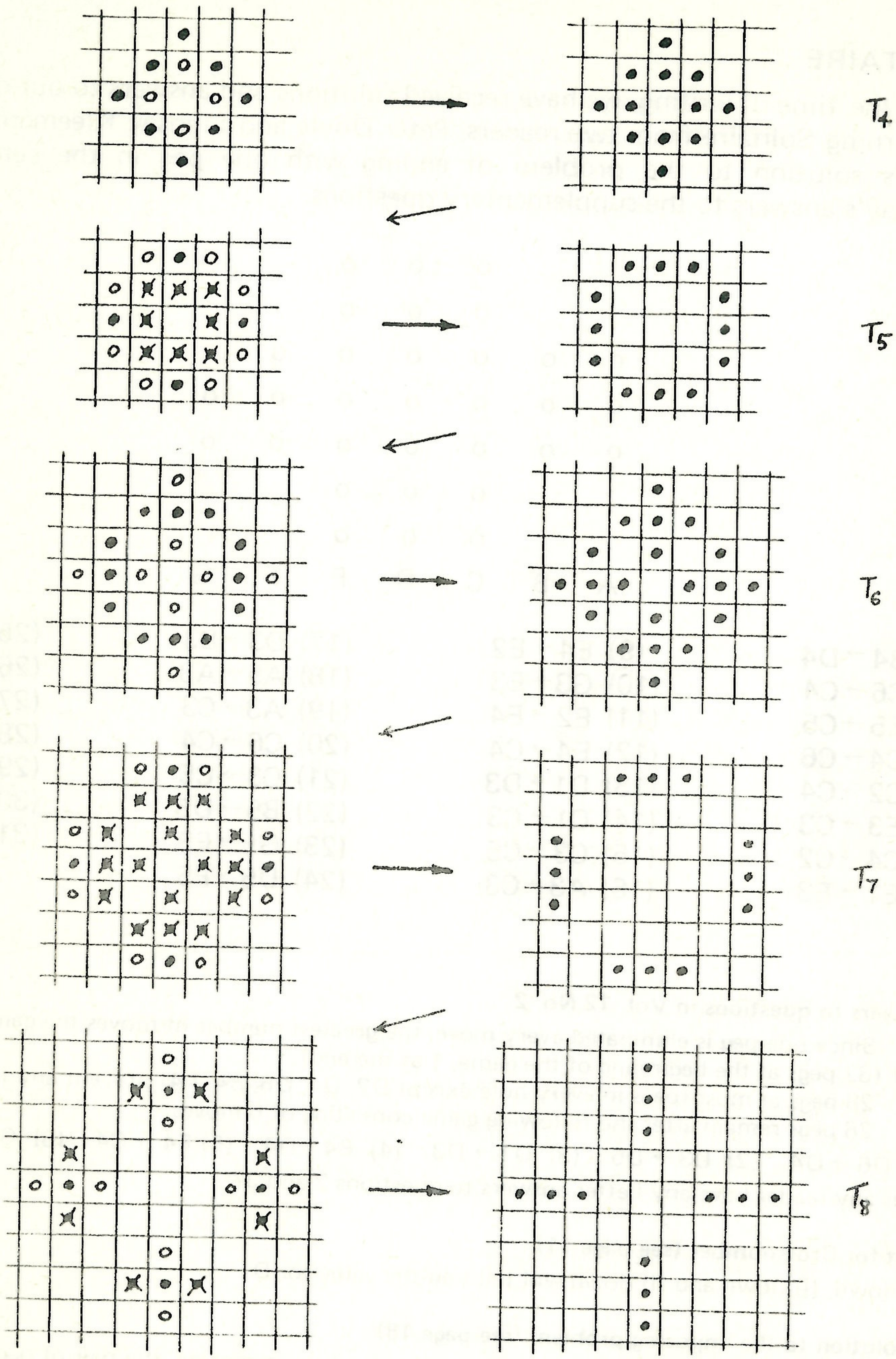
1. What is the least number of moves (by either player) a game can last?
2. If each player takes one marble each turn, the game will last 40 moves. However, if each player takes all the exposed marbles of his chosen colour at each turn, show that the number of moves must be less than 40. What is the maximum number of moves a game can last when this happens?
3. How many non-isomorphic arrangements of the marbles are possible at the start of a game? (Two arrangements are isomorphic if one can be changed into the other by re-arranging the rows or by re-arranging the colours of the marbles – such as swapping all the red marbles from green marbles.)
4. Can you find an arrangement of the forty marbles in which the best strategy is *not* always to remove all the exposed marbles of some colour?
5. Do you have any more suggestions for strategies?

LIFE

Since the last issue of Parabola (Vol. 12 No. 3), S. Anderson of Canberra Grammar School has sent in answers to all the questions we asked. To spur some N.S.W. readers on to try the other questions, I thought I should show you his answer to question 1 only. This question asks what happens to the population which starts life as "five in a line". Terms T_1 to T_8 are shown below. After this, $T_{7+2i} \equiv T_7$ and $T_{8+2i} \equiv T_8$ where i is a positive integer.

You might try other simple patterns for us to print in the next issue of Parabola.





T_4
T_5
T_6
T_7
T_8

SOLITAIRE

At the time of writing, we have received solutions and answers to our questions concerning Solitaire from two readers, Peter Doyle and Lindsay Kleeman. We give Peter's solution to the problem of ending with one peg in the centre, and Lindsay's answers to the supplementary questions.

		o	o	o			1
		o	o	o			2
o	o	o	o	o	o	o	3
o	o	o	o	o	o	o	4
o	o	o	o	o	o	o	5
		o	o	o			6
		o	o	o			7
A	B	C	D	E	F	G	

(1) B4 → D4
(2) C6 → C4
(3) E5 → C5
(4) C4 → C6
(5) C2 → C4
(6) E3 → C3
(7) C4 → C2
(8) E1 → E3
(9) E4 → E2
(10) G3 → E3
(11) E2 → E4
(12) E4 → C4
(13) D1 → D3
(14) C1 → C3
(15) C3 → C5
(16) A3 → C3
(17) D3 → B3
(18) A5 → A3
(19) A3 → C3
(20) C6 → C4
(21) C3 → C5
(22) B5 → D5
(23) G5 → E5
(24) D5 → F5
(25) E7 → E5
(26) G4 → E4
(27) E4 → E6
(28) C7 → E7
(29) E7 → E5
(30) F5 → D5
(31) D6 → D4

Answers to questions in Vol. 12 No. 2

1. Since one peg is eliminated every move, the greatest number of moves the game can have is 31 (32 pegs at the beginning of the game, 1 at the end).
2. 28 pegs at most (pegs in every hole except D2, D4, D6, B4, F4) prevent any further move.
3. 26 pegs remain after the following game consisting of 6 moves:

(1) D6 → D4 (2) D3 → D5 (3) D1 → D3 (4) B4 → D4 (5) E4 → C4 (6) G4 → E4.

Does any reader have any better answers to questions 2 and 3?

Hint for Crossnumber (see page 11)

(c) down, (j) down and (l) down will tell you the value for D.

A solution to the imposing problem (see page 18)

Given that page number 1 is on the right hand side of the magazine, the pair of pages are correctly imposed if and only if

n is odd, and
m + n = total number of pages + 1.