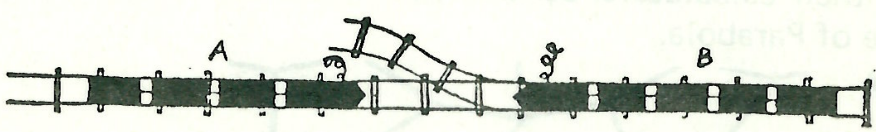
PROBLEM SECTION

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions, and the names of those who submit solutions before the publication of the next issue, will be published in Vol. 13 No. 2.

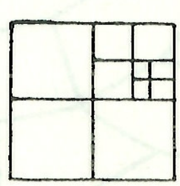
Although the problems are unclassified, the first 4 problems should require no more mathematical knowledge than is learnt in first or second form and the next 4 problems should require no more mathematical knowledge than is learnt in the first 4 forms of High School. Senior students are thus encouraged to attempt the later problems.

333. Two trains A and B are travelling in opposite directions on a line with a single track and wish to pass with the help of a siding (see figure).



The siding will only take one carriage or one engine at a time and can only be entered from the right. If train A has 3 carriages and train B has 4 carriages, how can they pass with the minimum number of moves?

334. It is impossible to dissect a square into 2 smaller squares, but it can be dissected into 10 smaller squares as shown in the figure.



Find all numbers n such that it is impossible to dissect a square into n squares.

335. A school held a special examination to decide which student in year 12 was best overall in the subjects English, History, French, Maths and Science. Five students – Alan, Barbara, Charles, David and Evonne – sat for five papers, one in each of these five subjects. To simplify matters, the top student in a paper was given 5 marks, the next student was given 4 marks, and so on, the last student in a paper being awarded 1 mark (fortunately, no two students tied in any of the papers). When the marks for each student were collected, the following facts were noted:

(1) Alan had an aggregate mark of 24;

(2) Charles had obtained the same mark in four out of the five subjects.

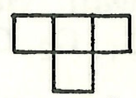
(3) Evonne, the Mathematician, had topped Mathematics, although she only came third in Science.

(4) The students' aggregate marks were in alphabetical order, and no two students had the same aggregate.

What we want to know is:

(a) What was Barbara's mark in Maths?

(b) How many of the 5 students obtained the same mark in at least four out of the five subjects? (Charles was one of these!)

336. You are given an 8 x 8 chessboard and 16 tiles of the shape  where each of the squares in the T-shape is the same size as the squares of the Chessboard.

(i) Can the Chessboard be completely covered with these tiles?

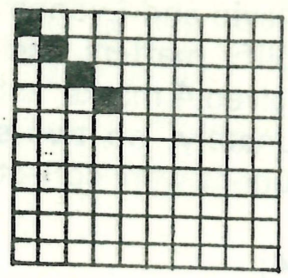
(ii) If one of the T-shaped tiles were replaced by a square tile which just covers four of the chessboard squares, can the chessboard be completely covered by these 16 tiles?

In each case, you must either show how to cover the board, or prove that it is impossible.

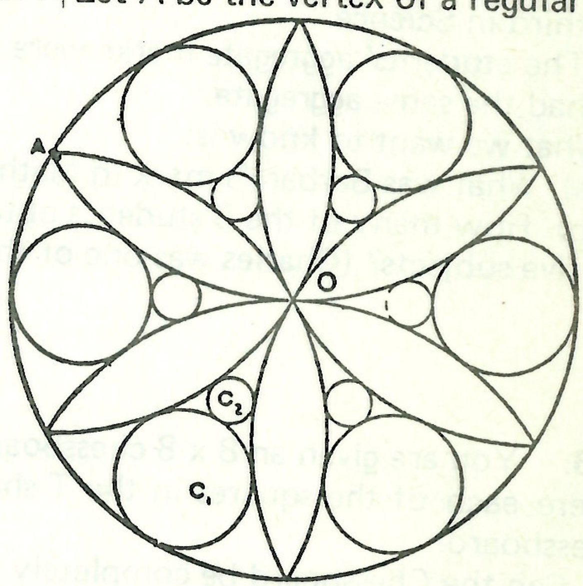
337. x and y are real numbers such that $x + y = 1$ and $x^4 + y^4 = 7$. Find $x^2 + y^2$ and $x^3 + y^3$.

338. You are given 216 blocks each of dimensions 1 cm x 1 cm x 8 cm. Is it possible to build a cube of dimensions 12 cm x 12 cm x 12 cm with these blocks? (As in problem 336, you must either show how to do it, or prove that it is impossible.)

339. Show how to dissect the square in the figure into 4 congruent pieces, each containing one of the black squares.



340. Let O be the centre of a circle C of radius r. Let A be the vertex of a regular hexagon inscribed in C. Using A and the other vertices of the hexagon as centres, arcs of radius r are drawn as in the figure. The result is the six-petaled "flower" of the figure. Next are drawn the largest circles which will fit between petals, for example C_1. Then is drawn the next largest C_2, and so on (remaining not drawn). What are the radii of the circles C_1, C_2, C_3, and so on?



341. A certain tribe of early men had an alphabet consisting of two letters A and B. They also had the rule that in any word ABA was equivalent with B (that is, each could replace the other in the word and the word was considered to be the same); and the rule that BAB was equivalent to A.

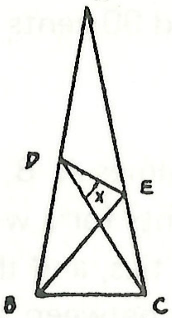
(i) How many different words could be represented?

(ii) Find two other ways of writing down a certain 4-lettered current "pop" group whose name may be written in this language.

342. Let S be the set of all points in the Cartesian plane whose coordinates (x,y) are both integers such that $0 \leqslant x \leqslant 100$, $0 \leqslant y \leqslant 100$. Show that however one chooses 5 points P_1, P_2, P_3, P_4, P_5 from S, at least one pair of these points has the property that the straight line through them contains a third point of S (possibly, but not necessarily, another of the chosen points). Does the statement remain true if 5 is replaced by 4?

343. We define a "shuffle" of a deck of N cards numbered 1, 2, . . . , N to be a specific procedure for arranging them in a different order. If one systematically repeats the same shuffle of the deck enough times, it returns to its original order. For instance, if the shuffle consists of interchanging the top 2 cards of the deck it returns to its original order after 2 shuffles. What shuffle of a deck of 28 cards requires the largest number of repetitions before returning to the original order?

344. *(Suggested by Mr G. Davis of North Sydney Tech. College).* In the following diagram *AB = *AC, $\angle DAE = 20°$, $\angle DCB = 60°$, $\angle EBC = 50°$ and $\angle CDE = x°$. Find x, without using trigonometric tables.



Solutions to Problems 321–332 (Vol. 12 No. 3)

321. The following factorisations of numbers are true:

$$12 = 3.4;\ \ 1122 = 34.33;\ \ 111222 = 334.333;\ \ 1111222 = 3334.3333$$

Can this scheme be continued indefinitely? Prove your answer.

Answer: Suppose A = 33 . . . 33 and B = 33 . . . 34 (each one having n digits). Then $3A = 99 \ldots 99 = 10^n - 1$, $3B = 100 \ldots 02 = 10^n + 2$, and

$$\begin{aligned} 9AB = (3A)(3B) &= (10^n - 1)(10^n + 2) \\ &= 10^{2n} + 10^n - 2 \\ &= 10^{2n} - 1 + 10^n - 1 \\ &= 99 \ldots 999 \ldots 99 + 99 \ldots 9 \end{aligned}$$

where the first number has 2n nines and the second has n nines. So AB = 11 . . . 111 . . . 1 + 11 . . . 1 = 11 . . . 122 . . . 2.