YOUR LETTERS

Dear sir,

My reason for writing is to find a method for getting the exact digits of π. Recently I have seen one method of getting it as

$$\pi = 4 \times (1 - 1/3 + 1/5 - 1/7 + 1/9 - \ldots).$$

Another has been

$$\pi = \tfrac{1}{2} \times \text{the circumference of a circle of radius 1},$$

which is not very helpful. The only other one I can find is an approximation by the perimeters of the inscribed and circumscribed regular polygons with n sides of a circle of radius 1:

n = 6	6.0	<	2π	<	6.8
n = 12	6.21	<	2π	<	6.43
n = 24	6.26	<	2π	<	6.32
n = 48	6.279	<	2π	<	6.292
n = 96	6.2821	<	2π	<	6.2854
n = 192	6.2829	<	2π	<	6.2837
n = 384	6.28311	<	2π	<	6.28333
n = 768	6.28317	<	2π	<	6.28322
n = 1536	6.28318	<	2π	<	6.283196
n = 3072	6.283184	<	2π	<	6.283188
n = 6144	6.283184	<	2π	<	6.283186

These are virtually the only solutions I can find, and would appreciate a bit of help.

Also, can you do a cheap cover for binding a few copies of Parabola?

Jimmy Pike

Editor's Reply:

With regard to your question on the value of π, you may be interested in some formulas used by various Mathematicians over the years:

(1) Using the fact that π is the circumference of a circle of radius 1, Archimedes showed that π is between 22/7 and 223/71.

(2) In the sixteenth century, Vieta produced "the first actual formula" (according to H.W. Turnbull):

$$2/\pi = \sqrt{\tfrac{1}{2}} \times \sqrt{(\tfrac{1}{2} + \tfrac{1}{2}\sqrt{\tfrac{1}{2}})} \times \sqrt{(\tfrac{1}{2} + \tfrac{1}{2}\sqrt{(\tfrac{1}{2} + \tfrac{1}{2}\sqrt{\tfrac{1}{2}})})} \times \ldots$$

(3) In the seventeenth century, an Englishman John Wallis showed that

$$\frac{2}{\pi} = \frac{3 \times 3 \times 5 \times 5 \times 7 \times 7 \ldots}{2 \times 2 \times 4 \times 4 \times 6 \times 6 \ldots}$$

and, using this result somehow, Lord Brouncker of Ireland gave the formula in Dr Loxton's article (Vol. 13 No. 1).

(4) Meanwhile, about the same time, a Scotsman called James Gregory noticed that if you integrate the series

$$1/(1 + x^2) = 1 - x^2 + x^4 - x^6 + \ldots$$

you get

$$\theta = x - x^3/3 + x^5/5 - x^7/7 + \ldots$$

where $\tan\theta = x$. By substituting $\theta = \pi/4$, Leibniz saw that

$$\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \ldots$$

since $\tan \pi/4 = 1$. This was such a major breakthrough that all subsequent evaluations of π have been based on it.

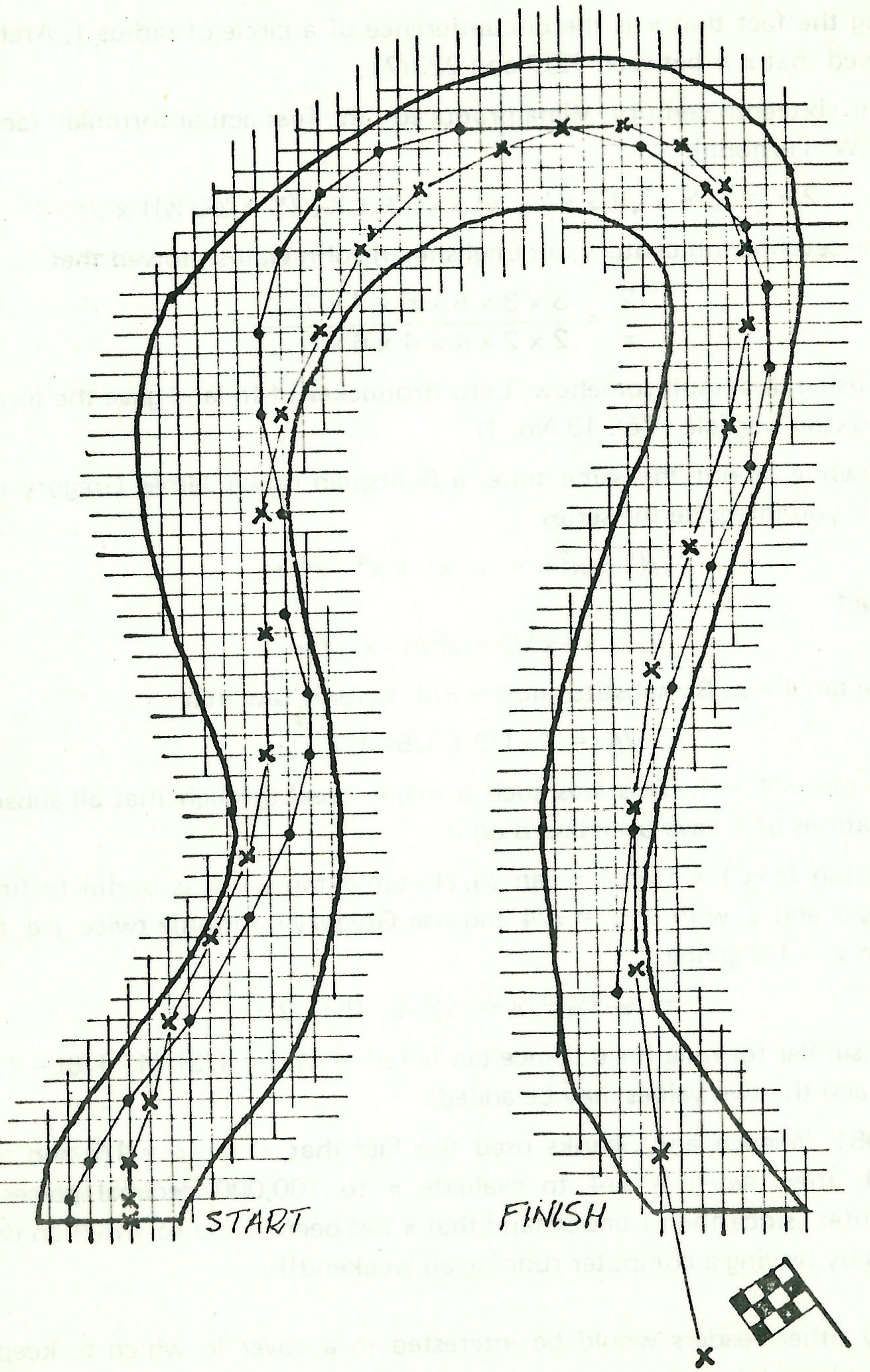
(a) Since $\tan(\theta+\psi) = (\tan\theta + \tan\psi)/(1 - \tan\theta\tan\psi)$, it is useful to find two angles θ and ψ with $\theta+\psi = \pi/4$ and use Gregory's formula twice, e.g. $\tan\theta = \tfrac{1}{2}$, $\tan\psi = 1/3$ giving

$$\theta = \tfrac{1}{2} - (\tfrac{1}{2})^3/3 + (\tfrac{1}{2})^5/5 - (\tfrac{1}{2})^7/7 + \ldots$$

and a similar formula for ψ. Since $\tan(\theta+\psi) = (1/2 + 1/3)/(1 - 1/6) = 1$, $\pi/4 = \theta+\psi$, and the two values may be added.

(b) In 1961, Wrench and Shanks used the fact that, if $\tan\alpha = 1/5$ and $\tan\beta = 1/239$, then $4\alpha - \beta = \pi/4$ to evaluate π to 100,000 decimal places on a computer (since then I understand that π has been found to 1 million decimal places by leaving a computer running all weekend!).

If any other readers would be interested in a cover in which to keep their Parabolas, please let me know.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