PROBLEM SECTION

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions, and the names of those who submit solutions before the publication of the next issue will be published in Vol. 13 No. 3.

Although the problems are unclassified, the first 4 problems should require no more mathematical knowledge than is learnt in first or second form and the next 4 problems should require no more mathematical knowledge than is learnt in the first 4 forms of High School. Senior students are thus encouraged to attempt the later problems.

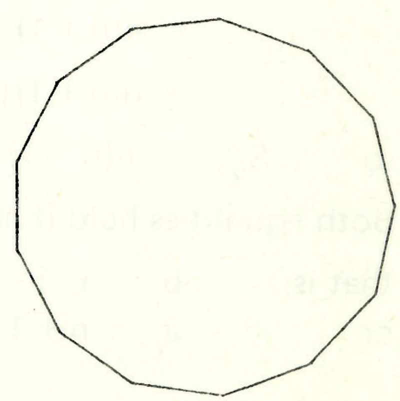
345. During a trial, three different witnesses A, B and C were called one after the other, and asked the same questions. In each case, each witness answered "yes" or "no", and the following facts were noted:

a) All questions answered "yes" by both B and C were also answered "yes" by A;
b) every question answered "yes" by A was also answered "yes" by B;
c) every question answered "yes" by B was also answered "yes" by at least one of A and C.

Show that the witnesses A and B agreed in their answers to all questions.

346. Five 2-digit numbers are made up from the digits 0, 1, . . . , 9, each digit being used exactly once. What is the largest possible value of the product of the five numbers?

347. Is it possible to select four vertices of the following regular 13-sided polygon so that the four sides and two diagonals of the quadrilateral formed by the four chosen vertices have different lengths?



348. Four explorers are going to make a trip into the desert. Each man can carry enough water to last ten days. Each man can walk 24 kilometres a day. Obviously if all four stay together they can manage a trip of only five days into the desert, leaving enough water to return. If our explorers are thinkers, how far can they manage to get into the desert before they have to return, assuming that the desert is so uninhabited that it is safe to leave water behind for the return trip, but no explorer can return to civilisation to replenish his supply and then return to the desert.

349. Solve the system of equations

$$x^{\log y} + y^{\log \sqrt{x}} = 110, \quad xy = 1000.$$

350. Let ABCD be a square of side 1 cm. Suppose P lies on BC, Q lies on DC, and that *AP = *AQ. Show that the perimeter of the triangle APQ is not more than $2 + \sqrt{2}$ cm.

351. A product $x \circ y$ is defined for all pairs of real numbers x,y so that the following hold for any x,y,z:

i) $x \circ y = y \circ x$

ii) $(x \circ y)z = xz \circ yz$

iii) $(x \circ y) + z = (x + z) \circ (y + z)$

What is the value of $99 \circ 100$?

352. How many square numbers are there whose digits, when written in base 10 notation, are three hundred 1's, and some number of 0's?

353. In travelling from A to B, a distance of 100 km, a train accelerates uniformly, travels 80 km at a constant speed of 100 km/h, and then decelerates uniformly. How long does the trip take?.

354. Prove that it is possible to select 2^k different numbers $a_1, \ldots a_{2^k}$ from the set $\{0, 1, 2, \ldots 3^k - 1\}$ in such a way that none of the a's is the arithmetic mean of any other two.

355. Let p be a prime number, and let $\binom{n}{p} = \frac{n(n-1)\ldots(n-p+1)}{1.2\ldots p}$, $[n/p]$ the largest integer not greater than n/p. Prove that $\binom{n}{p} - [\frac{n}{p}]$ is divisible by p.

356. A suitor asking for the hand of the king's daughter is given the following task:

Divide the square wall of the princess's room into ten smaller squares, a different way on each day for a week. No square should have the same 4 vertices as any square used on previous days. Is it possible for the suitor to marry the princess, or will he end up on the chopping block?

Late solver to Problems 321-332

M. Reynolds (Marist Brothers, Pagewood) 329.

Solutions to Problems 333 – 344 will be printed in the next issue and you are still invited to send your solutions to these problems as well.

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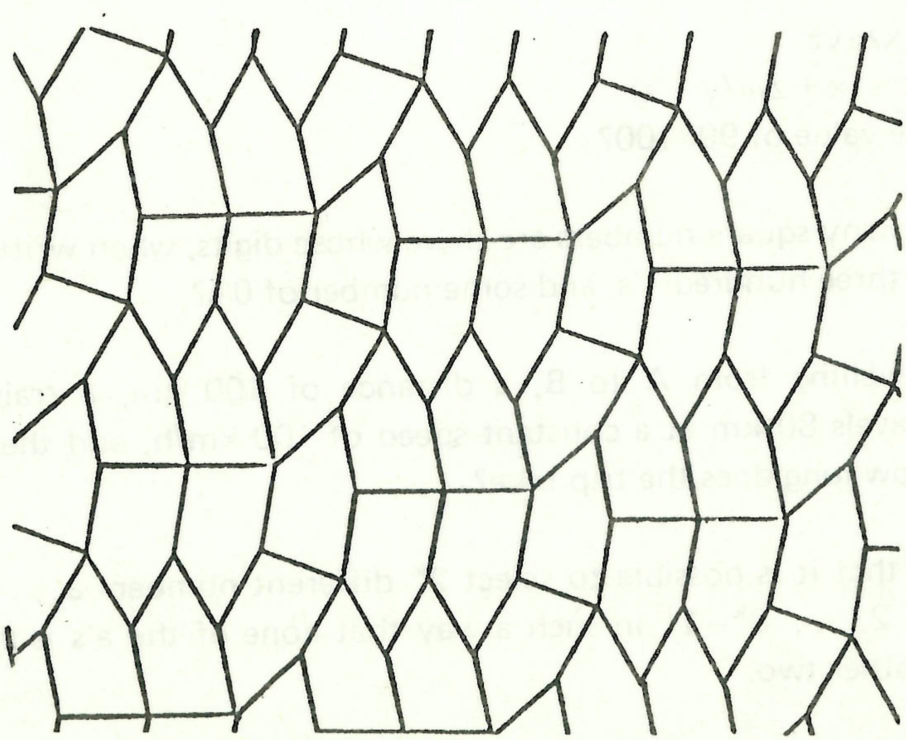

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