Parabola Volume 13, Issue 3 (1977)

SCHOOL MATHEMATICS COMPETITION, 1977
EXAMINERS' COMMENTS

(see Vol. 13 No. 2 for problems and solutions)

Senior Division

Generally speaking, the performance of entrants was disappointing. It has been put to me that the problems were too difficult, and I am prepared to concede that question 4, the one on sequences, may have been. However, I would make the point that every question was answered correctly by someone.

It is bad enough when even one entrant says something along the lines

" $$2^{7a} + 2^{9b} = 2^{8c},$$

so, taking logarithms,

$$7a + 9b = 8c$$ ",

but when scores of entrants do so, it is truly deplorable.

There were several high-points among the solutions. I mention two.

The first, due essentially to Alexander Anderson (Canberra Grammar) who pointed out that question 1(ii) has a solution in powers of 2:

Let $w = 2^{30 + 168t}$, $x = 2^{8473 + 47448t}$,

$y = 2^{6590 + 36904t}$, $z = 2^{7414 + 41517t}$.

Then $w^{1977} + x^7 + y^9 =$

$$= 2^{59310 + 332136t} + 2^{59311 + 332136t} + 2^{59310 + 332136t}$$

$$= 2^{50310 + 332136t} \times (1 + 2 + 1)$$

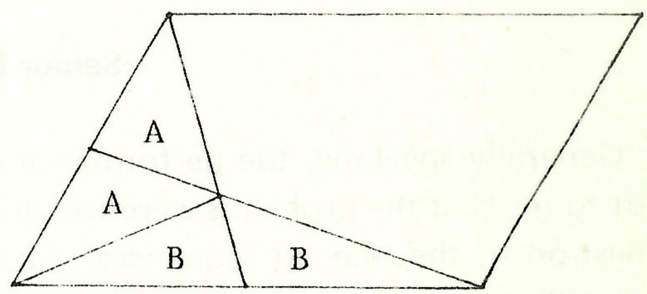
$$= 2^{59312 + 332136t} \quad \text{since } 1 + 2 + 1 = 4 = 2^2.$$

$$= 2^8(7414 + 41517t)$$

$$= z^8.$$

The second, due to Raymond Silins (Marsden High), who gave the following beautifully simple solution to question 3:

The two triangles marked A have equal areas, because they have equal bases and perpendicular heights: similarly for the two triangles marked B. If the parallelogram shown has area Q, then we can write



$$2A + B = Q/4$$

and

$$A + 2B = Q/4$$

Adding, we obtain

$$3A + 3B = Q/2$$

or,

$$A + B = Q/6$$

The result follows.

(It is also true that A = B. Can you see how to deduce this from the above equations?)

In conclusion, I wish to congratulate all the prizewinners.

Michael Hirschhorn

Junior Division

Question 1. Quite a few contestants got this right by an experimental process as follows:-

Base = 6, $m^2 - 11m + 52 = 32$

Base = 7, $m^2 - 11m + 52 = 30.$

So each increase of 1 in the base number gives a decrease of 2 in the value of $m^2 - 11m + 52$. Therefore base number = say, 7 + ½(30) = 22.

By the way, Julia Roberts of Ascham, getting No. 1 right, said that the Atlanteans obviously had 22 as the base of their number system because they had 11 fingers on each hand, just as we use base 10 because we have 5 on each hand. We never thought of that – good on you, Julia.

However most students forgot the other answer to the quadratic equation:- $m = 16$ (to base 10).

Question 2. (i) This was probably the most successfully attempted question. Many got the right idea that the maximum value of the product of two rational or real numbers would come from putting them equal so that each = ½ x 1,977 – but the question said "natural numbers" (positive integers) so that you had to take one as ½ x 1,977 + ½ and the other as ½ x 1,977 – ½. But I didn't deduct very much for that mistake.

(ii) Students saw that generally you wanted as many numbers as possible to multiply together, although 1,977 1's is no good.

A 1 and 988 2's is better, a 3 and 987 2's is better still, and 659 3's is best of all.

Question 3. Tiling a plane means completely covering it – but, of course, the plane is also infinite so perhaps that's a contradiction! An adequate explanation or calculation plus diagram was essential here for full marks. I think some of the unsuccessful thought they could cut the plane up first to suit themselves.

Question 4. Some read this question "3 contestants + 1 winner = 4 players". If they got a right answer on that basis they got over half-marks. The main weakness of those who understood the question was a failure to consider all the possible alternatives so as to be able to pick the **greatest** number possible to fit the circumstances. It was a question well suited to solution using a diagram as in "Parabola" and most contestants attempting it used this approach.

Question 5. Very few students indeed even got started, because they obviously didn't understand what the question wanted. Anybody who showed any understanding got **some** marks even though they then got it quite wrong.

General. As this is NOT an ordinary examination, quite high marks are always possible without getting all the answers numerically right. What one is looking for is an ability to think about the question, an ability to adopt or invent a logical method for dealing with it – and a readiness, sometimes, to stand on your (mathematical!) head to see what happens if you look at the problem in an unusual way. The only way an examiner can pick this up is from what you put down in the exam books; ideas kept in your head remain invisible to him. In this connection, even "messy" arithmetical calculation is important to the examiner if only to know which way the student got the number concerned. If you do a "sum" on your calculator, say what the "sum" is. As far as the examiner is concerned, in this Competition "rough" work is never shameful, should always be handed in. Try and tell us, in your answers, what you were doing, how you were doing it and why you were doing it.

I say this because every year I feel sure that some candidates were really worth quite a few more marks than I could give them on the evidence of their written answers. There are tantalising hints in a phrase used or a calculation mentioned, to suggest the competitor might really know the right method and have used it – but no straightforward evidence!

This is my last year as examiner in the School Mathematics Competition. I must have been marking the Junior Section for 10 or 11 years now. It has been a cheering experience because it has showed the number of junior school students of both sexes who find pleasure in solving maths problems; and I would hope that the Competition and Parabola have made them feel a little less lonely in their unusual enthusiasm! Long may both continue!

Signing off,

Michael Greening

SOLUTION TO CROSSNUMBER PUZZLE (Vol. 13 No. 1)

Solved by K. Lam (Year II, Trinity Grammar),
and P. Crump (Year II, Sydney Grammar).

	a 3	b 7	c 4		
d 1		e 1	9	f 1	
g 1	h 4	4		i 2	j 8
k 2	9		l 1	2	2
	m 1	n 2	0		2
		o 4	5	1	