PROBLEM SECTION

You are invited to submit solutions to one or more of these problems. Answers should bear your name, year, and school. Solutions will be published in the issue after next.

369. Find a five digit number which when divided by 4 yields another 5 digit number using the same 5 digits but in the opposite order.

370. Prove that, when $x > 0$, $(1 + x^2 + x^4)/(x + x^3) \geq 3/2$.

371. A game is played by two players with matchsticks, as follows. To start, 36 matches are equally spaced in a row: . . . . Each player picks up in turn, either one, two, or three matches. The player who picks up the last match wins the game.

(a) Prove that the second player can always win.

(b) The rules are changed, to require that the one, two, or three matches must be neighbouring matches from one group. Thus, after the first player's turn the remaining matches are in one or two groups. The second player must choose his matches, all together, from one of these groups. There will then be one, two, or three groups, from one of which the first player must pick his matches, etc.
Can the second player still always win? Prove your answer.

372. After the first day of classes, each of 5 different students knows a different bit of gossip about the teachers in their school. When they get to their separate homes, the telephoning begins. Assume that whenever anyone calls anyone else, each tells the other all the gossip he knows. What is the smallest number of calls after which it is possible for every student to know all 5 bits of gossip?

373. For which values of n is $1^n + 2^n + 3^n + 4^n$ divisible by 5?

374. *(a)* A plane figure has one axis of symmetry and a point on that axis is a centre of symmetry. Does the figure necessarily have a second axis of symmetry?

(b) A 3-dimensional figure has one plane of symmetry, and a point in that plane is a centre of symmetry. Does the figure necessarily have a second plane of symmetry?

375. A cornfield has 1000 cornstalks. When the farmer stands at a cornstalk at the corner of the field he notices that some of the cornstalks line up with the one he is standing at. On closer examination it turns out that the number of these lines which contain an odd number (such as 1, 3, 5, . . .) of other cornstalks is odd. Is this true no matter which cornstalk he stands at? Why or why not?

376. Given n sacks each holding the same number of apples. On the first day an apple is removed from one sack. On the second day an apple is removed from each of 2 sacks; and so on until the nth day when one apple is removed from each of the n sacks.

The sacks are now all empty. For which n is this possible, and how is it to be done?

377. Let x_i, y_i ($i = 1, 2, \ldots n$) be real numbers such that $x_1 \geq x_2 \geq \ldots \geq x_n$ and $y_1 \geq y_2 \geq \ldots \geq y_n$.

Prove that, if $z_1, z_2, \ldots, z_n$ is any rearrangement of $y_1, y_2, \ldots, y_n$ then

$$\sum_{i=1}^{n} (x_i - y_i)^2 \leq \sum_{i=1}^{n} (x_i - z_i)^2$$

378. Let $a_1, a_2, a_3 \ldots$ be any infinite sequence of strictly positive integers such that $a_k < a_{k+1}$ for all k. Prove that infinitely many a_m can be written in the form $a_m = xa_p + ya_q$ with x, y strictly positive integers and $p \neq q$.

379. On the sides of an arbitrary triangle ABC, triangles ABR, BCP and CAQ are constructed externally with $\angle PBC = \angle CAQ = 45°$; $\angle BCP = \angle QCA = 30°$; $\angle ABR = \angle RAB = 15°$.

Prove that $\angle QRP = 90°$ and QR = RP.

380. When 4444^{4444} is written in decimal notation, the sum of its digits is A. Let B be the sum of the digits of A. Find the sum of the digits of B.

Solutions to Problems from Vol. 13 No. 2

345. During a trial, three different witnesses A, B and C were called one after the other, and asked the same questions. In each case, each witness answered "yes" or "no", and the following facts were noted:

a) All questions answered "yes" by both B and C were also answered "yes" by A;
b) every question answered "yes" by A was also answered "yes" by B;
c) every question answered "yes" by B was also answered "yes" by at least one of A and C.

Show that the witnesses A and B agreed in their answers to all questions.