THE VARYING EFFECT OF MORTALITY

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This article looks at some of the mathematics used by actuaries, and how it describes some features of our lives.

Rates of Mortality, Probabilities of Survival and the Life Table

The chance of a person age x dying between age x and x + 1 is denoted by q_x, while the probability of a person surviving from age x to age x + 1 is denoted by p_x, that is $p_x = 1 - q_x$.

The probability of a person surviving from age x to x + t is denoted by ${}_tp_x = p_x . p_{x+1} . p_{x+2} \ldots p_{x+t-1}$. In order to use these probabilities more conveniently, actuaries use a hypothetical model called a Life Table. It commences with an arbitrary large number of lives at a convenient age (usually 0, but sometimes other ages are used). The model shows the number ℓ_x at each age x surviving from the original number, assuming the lives at each age experience mortality according to some scale of mortality.

$$q_x = (\ell_x - \ell_{x+1})/\ell_x$$
$$p_x = \ell_{x+1}/\ell_x$$
$${}_tp_x = \ell_{x+t}/\ell_x$$

At the age (ω) by which all lives are assumed to have ended,

$$\ell_\omega = 0$$

As mortality is a probability relating to the average of a particular group of lives, mortality varies from group to group. For example, males and females in the Australian population exhibit different mortality, and this varies with time. The Australian Government Actuary uses $\ell_0 = 100{,}000$ for the Australian Life Tables. Using mortality measured over the years indicated the Australian Life Tables give the following values of ℓ_x.

	Years 1901-10		Years 1970-72	
Age	Males	Females	Males	Females
0	100,000	100,000	100,000	100,000
20	84,493	86,459	96,473	97,596
40	75,887	78,001	93,150	95,848
60	56,782	63,247	77,574	86,719
80	14,330	21,356	23,399	44,242

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This table indicates how dramatically mortality has reduced since the turn of the century, and also the greater longevity of females.

Expectation of Life

The average duration of life for a member of a group of lives is known as the "expectation of life". This can be calculated starting at any age, although "expectation of life" as commonly used means from birth. The actuarial formula is —

$$\text{Expectation of life} = (\tfrac{1}{2} + \ell_1 + \ell_2 + \ldots)/\ell_0$$

$$\text{Expectation of life for those alive at age x} = (\tfrac{1}{2} + \ell_{x+1} + \ell_{x+2} + \ldots)/\ell_x$$

From the examples from the Australian Life Tables, expectations are:-

	1901–10		1970–72	
Age	Males	Females	Males	Females
0	55.20	58.84	67.81	74.49
40	28.56	31.47	31.61	37.16
80	4.96	5.73	5.52	6.88

Curve of Deaths

The number of deaths at age x is denoted in the life table as d_x, so that

$$d_x = \ell_x - \ell_{x+1} = \ell_x q_x$$

As all lives must die by age ω,

$$\sum_0^{\omega} d_x = \ell_0 = 100{,}000$$

The following graph shows the number of deaths at each age for males and females according to the 1970-72 Australian Life Tables.

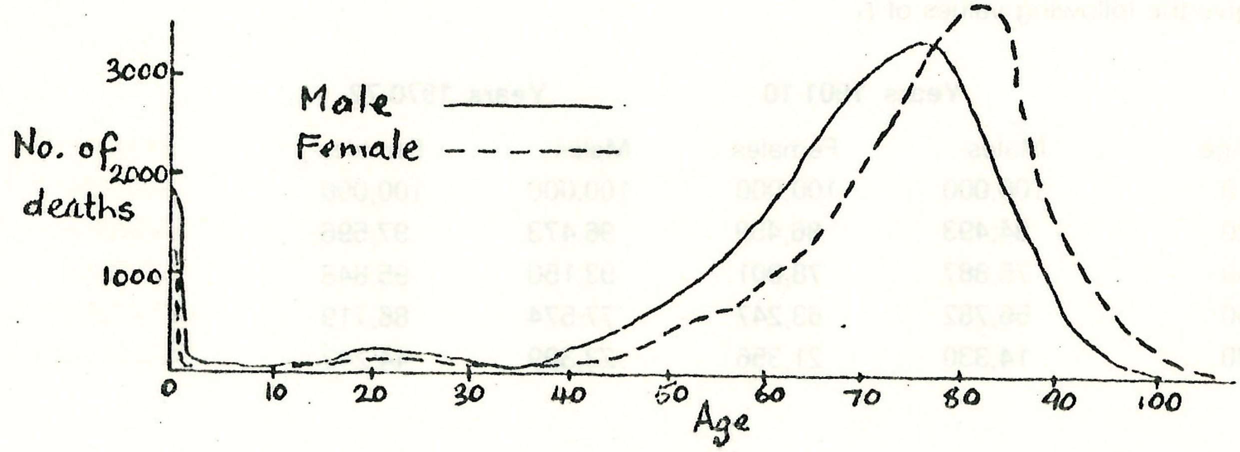

Figure 1

The total area under each of these curves is the same, as each shows the deaths from 100,000 births. The lower mortality rates for females cause the years at which the peak number of deaths occur to be later than the peak for males. This difference accounts for the fact that expectation of life at birth for males (67.8 years) is less than that for females (74.5 years).

Annual mortality rates at young ages are fairly small. Thus both male and female rates reach 1% between 50 and 60. If the male rates of mortality were doubled at ages above 30 the expectation of life would decrease from 67.8 years to 61.1 years. The curve of deaths would change in the following manner:

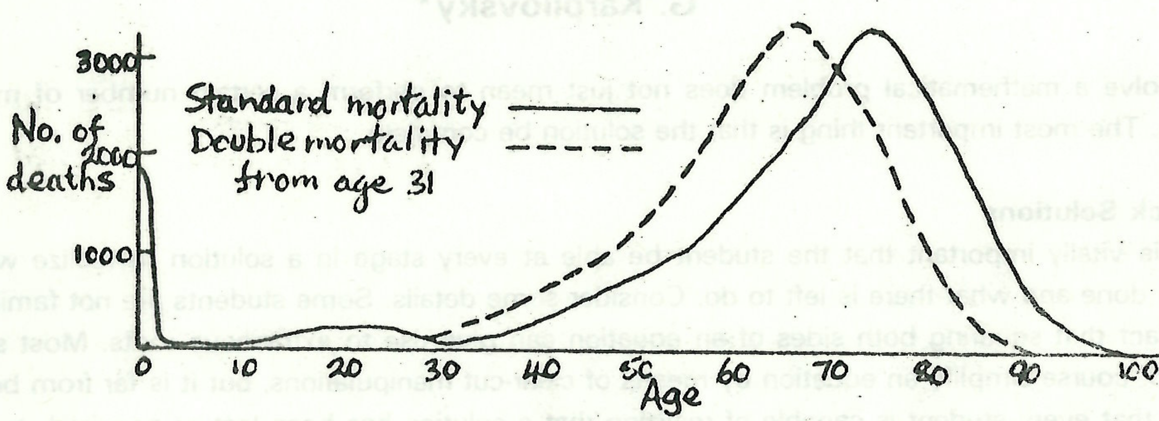

Figure 2

This difference is believed to correspond roughly to the effect of being a heavy smoker (30–40 cigarettes a day), rather than being a non-smoker.

A problem for the reader: Can you explain how the formula for "expectation of life" is derived?

TRY THIS FOR SIZE

In Vol. 14 No. 1 we asked the following questions:

How many digits does 1001^{1000} have?

How many can you find?

Ken Kerr, year 7, North Sydney Boys' High was able to tell us that 1001^{1000} has 3001 digits. Here are two hints to help you with the second question.

(i) $1001^{1000} = 10^{3000} \times (1 + (1/n))^n$, where $n = 1000$

(ii) $1001^{1000} = (1 + x)^P$, where $x = 1000$, $P = 1000$.