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THE MYSTERIES OF ANCIENT INDIAN MATHEMATICS

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§1. Indian Magic Squares

Several thousand years ago, magic squares were being used by Indian astrologers in their calculations of horoscopes. At the famous temples at Khajuraho in India there is still to-day an inscription of the following magic square:

9	16	2	7
6	3	13	12
15	10	8	1
4	5	11	14

With every row and column and both diagonals adding up to 34 this is one of the simplest magic squares. The key to their construction is found in the Indian scriptures called **Vedas**, written around 1000 to 600 B.C.

In casting a horoscope, the astrologer constructs the magic square according to a simple code. The construction is based on a number (any number) which is to be the sum of any of the rows, columns or major diagonals. Thus if one chooses the number 100, (note that any number will do as we shall see later), then the magic square will be

42	49	2	7
6	3	46	45
48	43	8	1
4	5	44	47

Now not only does every horizontal, vertical and diagonal row add up to 100, so also do the 2×2 squares at each of the corners and the 2×2 centre.

What other groups of four numbers add up to 100?

How is the square constructed? Let's assume that you want to construct the magic square for a

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friend... First you must ask for a number, any number. Let this number be S. The magic square will look like this:

$\frac{1}{2}S-8$	$\frac{1}{2}S-1$	2	7
6	3	$\frac{1}{2}S-4$	$\frac{1}{2}S-5$
$\frac{1}{2}S-2$	$\frac{1}{2}S-7$	8	1
4	5	$\frac{1}{2}S-6$	$\frac{1}{2}S-3$

Check that every row and column adds up to the number S.

Although we know how to construct such magic squares, their **interpretation** is still a mystery.

§2. The origin of our Number System

Ten has always formed the basis of numeration in India. While the Greeks had no names for numbers above the Myriad (10^4) the Hindus had names for numbers up to the Asankhyeya (10^{140}). To this day the Indian numeral language is more developed than that of any other nation.

Although the Hindus may not have invented zero (it is thought to have been first used in Babylon), the early mathematicians in India were the first to develop the arithmetic and algebraic manipulations with it.

At first the numbers 0 to 9 were represented by letters of the alphabet. This was done, not so that knowledge could be concealed, but because the text books could be written in verse for easy memorisation. Therefore each verse could have two different meanings. For example, it is claimed that a particular hymn to Lord Krishna is so worded as to give the correct evaluation of π to 32 decimal places with a self-contained key for extending the valuation to any number of decimal places! (This was discovered by Dr V.P. Dalal of Heidelberg University.)

The final development of the decimal system as we know it came with the use of symbols to represent numbers. Historians of mathematics have considerable evidence that the so-called "Arabic" numerals are of Hindu origin. Because of the strong trading links between the Hindu people and the Arab people, there was also an interchange of knowledge and ideas. It was in this way that the Hindu number system was adopted by the Arabs, and subsequently the Europeans.

After developing this numeral system, the Indians went on to create the system of arithmetic operations: addition, subtraction, multiplication and division. For some of these operations, their techniques were not only different from the ones that we normally use, but also in some cases more efficient. As an example I will explain their technique for writing down the product of two large numbers in one line.

§3. One-Line Answers to Arithmetic Problems

One-line solutions were contained in short formulae called **Sutras**. As an example, one such Sutra means "vertically and crosswise." This is a formula which can be applied in the following way to a multiplication problem:

Example 1

To evaluate 23 × 13:

$$\begin{array}{r} 23 \\ 13 \\ \hline 299 \end{array}$$

(i) We multiply vertically the first digits on the right of the multiplier and the multiplicand (3 × 3 = 9)

(ii) we multiply crosswise, 2 × 3 and 1 × 3, and add (6 + 3 = 9),

(iii) we multiply vertically the digits on the left (2 × 1 = 2).

(N.B. If one of the results of multiplication contains more than one digit we simply carry the left most digit over to the left.) Hence the result: 23 × 13 = 299.

The algebraic principle behind this can be represented in the multiplication of two polynomials; (ax + b) and (cx + d); where in the above example x = 10, a = 2, b = 3, c = 1 and d = 3:

$$(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd.$$

Thus, for the solution, the digit on the right is the product bd, the central digit is (ad + bc) and the digit on the left is ac.

This method can be used to write down in one line (this comes with a little practice) the answers to time-consuming multiplication problems:

Example 2.

To evaluate 532 × 472:

$$\begin{array}{r} 532 \\ 472 \\ \hline 251104 \end{array}$$

(i) We multiply vertically the right hand digits (2 × 2 = 4),

(ii) we multiply crosswise 3 × 2 and 7 × 2 and add (6 + 14 = 20) so we write 0 and carry 2,

(iii) we multiply vertically 3 × 7 and crosswise 5 × 2 and 4 × 2 and add (21 + 10 + 8 = 39) plus the carried 2 gives 41, and we carry 4,

(iv) we multiply 5 × 7 and 4 × 3 and add (35 + 12 = 47) plus the carried 4 gives 51, and we carry 5,

(v) we multiply 5 × 4, add the carried 5, gives 25.

Try justifying the one-line method for multiplying two 3-digit numbers algebraically by using the two polynomials $ax^2 + bx + c$ and $dx^2 + ex + f$. In contrast, the multiplication method that we would normally adopt takes several more lines:

$$\begin{array}{r} 532 \\ 472 \\ \hline 1064 \\ 37240 \\ 212800 \\ \hline 251104 \end{array}$$

For practice, use the one-line method to calculate 231 × 192 and 41 × 718.

These ancient techniques for multiplication can be extended to find cubes, fourth powers and so on.

Example 3.

Find 13^3. $13^3 = 2197$.

Algebraically, the solution is found from:

$$(ax + b)^3 = a^3x^3 + 3a^2bx^2 + 3ab^2x + b^3$$

where $x = 10$ and in this case $a = 1$ and $b = 3$.

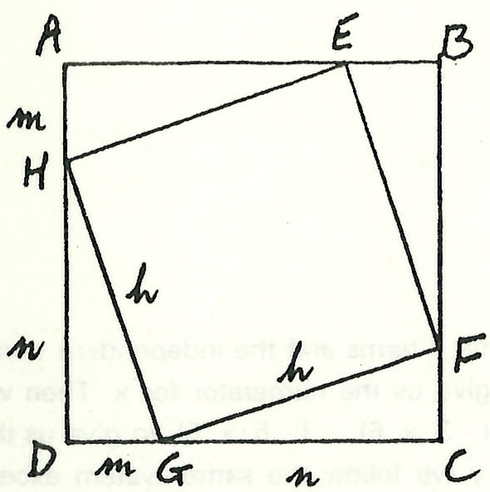
These techniques, and their further applications to long division and the finding of square roots, cube roots, and so on were well developed at a time when the Greeks were struggling with the abacus.

§4. The Theorem of the Square on the Hypotenuse.

According to Indian historians of mathematics, the theorem of the square on the hypotenuse was not discovered by Pythagoras. Although it is now called the Theorem of Pythagoras it was discovered in India before Pythagoras was born.

Of the five different proofs that were developed by the Indian ancients, two will be outlined here:

Proof 1.

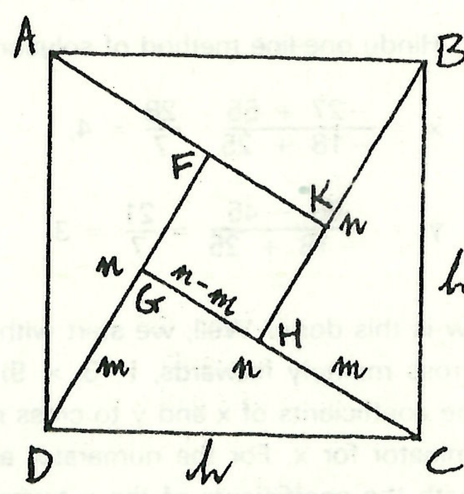


area ABCD = 4 × area EBF + area EFGH

$\therefore (m+n)^2 = 4 \times (\frac{1}{2}mn) + h^2$

$\therefore h^2 = m^2 + n^2$

Proof 2.



area ABCD = 4 × area ABK + area FGHK

$\therefore h^2 = 4 \times (\frac{1}{2}mn) + (n-m)^2$

$\therefore h^2 = m^2 + n^2$.

These proofs are given in the **Sulba-sutras** (sulba = measurement, sutra = formula) of the Vedas. These ancient texts in geometry gave the principles and procedures for the construction of altars and temples in different geometrical shapes.

These texts also refer to important properties of circles and triangles which show that most of the theorems in the books of Euclid (the ancient Greek who is often considered to be the father of geometry) must have been worked out before his time.

§5. The Science of Calculation with Unknowns

Arithmetic and geometry were referred to as the "science of calculation with knowns", while algebra was called the "science of calculation with unknowns".

Early developments in Indian algebra we will illustrate with one example for which the method is quicker than for the techniques we normally use.

The Solution of Simultaneous Equations

Example 1. Solve:

$$5x - 3y = 11, \quad (1)$$

$$6x - 5y = 9. \quad (2)$$

a) Normal method of solution:

(1) × 6: $30x - 18y = 66$ (3)

(2) × 5: $30x - 25y = 45$ (4)

Subtract: $\therefore 7y = 21$

$\therefore y = 3$

put $y = 3$ into (1): $5x - 9 = 11$

$\therefore x = 4$

b) Hindu one-line method of solution:

$$x = \frac{-27 + 55}{-18 + 25} = \frac{28}{7} = 4,$$

$$y = \frac{66 - 45}{-18 + 25} = \frac{21}{7} = 3.$$

How is this done? Well, we start with the coefficients of the y terms and the independent terms and cross multiply forwards, $(-3 \times 9) - (-5 \times 11)$, to give us the numerator for x. Then we use the coefficients of x and y to cross multiply backwards, $(-3 \times 6) - (-5 \times 5)$, to give us the denominator for x. For the numerator and denominator for y we follow the same system except that with the coefficients of the x terms and the independent terms we cross multiply backwards $(6 \times 11) - (5 \times 9)$; the denominator being the same as for x.

Can you solve the following two simultaneous equations using this method?

$$11x + 6y = 28,$$

$$7x - 4y = 10.$$

(Answer: $x = 2$, $y = 1$).
Can you show that this method works?

§6. The Lost Tradition

The word **Veda** means "storehouse of all knowledge". In theory then, the Vedas should contain all knowledge relating to mankind. They were written in four separate parts:

1) Ayur-Veda (anatomy, physiology, medicine)
2) Dhan-Veda (archery, military science)
3) Gandharva-Veda (art and science of music)
4) Sthapatya-Veda (architecture, astronomy, mathematics)

If one travels in India to-day one finds that many of the practising doctors work in the Ayur-Veda tradition. They still refer to the ancient texts for information on the treatment of illnesses. Modern medicine is only now discovering how effective these traditional remedies are.

In the case of mathematics, there is no longer a living tradition based on the Vedas. The decline of Hindu mathematics (by the 12th century A.D.) was partly the result of foreign invasions and internal warfare, and partly due to the rise of astrology. Professional fortune-tellers only needed enough mathematics and astronomy to impress their clients.

In the Western world it is generally thought that the ancient Greeks were the first mathematicians. While this may be true as far as a deductive method is concerned, that is, a method based on proofs and axioms, there is no doubt that in the **application** of mathematics, in arithmetic, geometry and algebra, the Indians had a clear lead over the contemporary Greek mathematicians.