BRICKS THAT (ALMOST) TOPPLE

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Can you identify the next term of the series: 1, 5, 32, 228, 1675, ?
This is probably a little hard without knowing how the first numbers were generated.
Imagine 2 bricks, one on top of the other, but protruding half a brick.

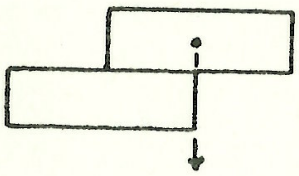

Figure 1

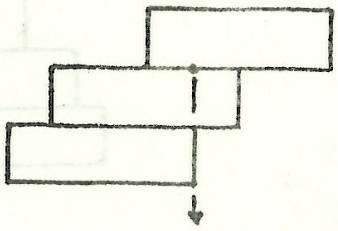

Figure 2

The centre of gravity (COG) of the top brick passes through the right-hand vertical face (RHVF) of the bottom brick giving a (just) stable pile. Imagine now if we lift the 2 bricks and slide another under the bottom brick in such a position that the COG of the 2 top bricks passes through the RHVF of the now bottom brick.

The pile is obviously still stable.

It is amazing that this process can be repeated indefinitely and that with a stack of 5 bricks we have the top brick completely overhanging the bottom brick! Yet the pile is stable. If we continue on in this fashion, by the time we have reached 32 bricks, the top brick overhangs the bottom brick by 2 full bricks. To get a 3 brick overhang takes 228 bricks, while a 4 brick overhang takes 1,675 bricks and a 5 brick takes a mammoth 12,368 bricks. This is the answer to the question posed at the beginning of this article.

The bricks and their overhangs are shown in the following table:

*Mr Joyce is an adult reader of Parabola.

No. of Bricks	Overhang
1	0
5	1.041666667
32	2.013622598
228	3.002183353
1,675	4.000242778
12,368	5.000021437

The method of calculation was the formula below and a small programmable hand calculator. The calculation of the 5 brick overhang took over 2 hours running time.

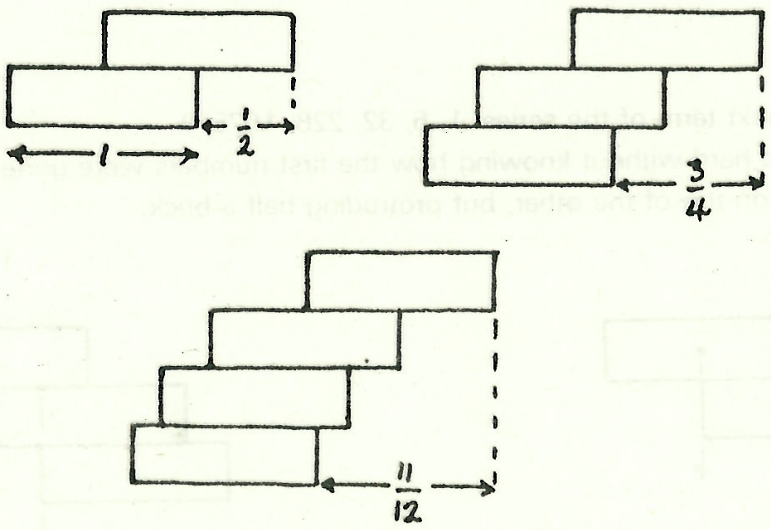

Figure 3

The overhang 0(n) of a stack of n+1 bricks is given by the formulae:

$$0(1) = 1/2, \quad 0(n) = 0(n-1) + 1/(2n),$$

or, by

$$0(n) = (1/2) + (1/4) + \ldots + (1/(2n)).$$

Thus,

$$0(1) = 1/2, \quad 0(2) = 3/4, \quad 0(3) = 11/12$$

(see figure 3).

Perhaps some reader with a faster calculator could calculate the number of bricks required for an overhang of 6. I estimate it to be about 91,500.

[Can any reader prove Mr Joyce's formula for 0(n)? Please send us your proof!]